The External Network Problem and the Source Location Problem

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The connectivity of a communications network can sometimes be enhanced if the nodes are able, at some expense, to form links using an external network. Using graphs, the following is a possible way to model such situations.

Let $G = (V, E)$ be a graph. A subset X of vertices in V may be chosen, the so-called *external vertices*. An *internal path* is a normal path in G; an *external path* is a pair of internal paths $P_1 = v_1 \cdots v_s$, $P_2 = w_1 \cdots w_t$ in G such that v_s and w_1 are from the chosen set X of external vertices. (The idea is that v_1 can contact w_t along this path using an external link from v_s to w_1.) For digraphs we use similar vocabulary, but then using directed paths.

Next suppose a certain desired connectivity for the network is prescribed (in terms of edge, vertex or arc-connectivity). Say that for a given k there need to be at least k edge- (or vertex- or arc-) disjoint paths (which can be internal or external) between any pair of vertices. What is the smallest set X of external vertices such that this can be achieved ?

A related problem is the Source Location Problem : In this we need to find, given a graph or digraph and a required connectivity requirement, a subset S of the vertices such that from each vertex in the graph there are at least the required number of edge- (or vertex- or arc-) disjoint paths between any vertex and the set S. And again, the goal is to minimise the number of vertices in S.

It seems clear that the External Network Problem and the Source Location Problem are closely related. In this talk we discuss these relations, and also show some instances where the problems behave quite differently. Some recent results on the complexity of the different types of problems will be presented as well.