A graph theoretic generalization of the number partition problem

Jørgen Bang-Jensen, Christoffer Christensen and Inge Li Gørtz
Department of Mathematics and Computer Science, University of Southern Denmark
Daniel Goncalves, LaBRI, Université Bordeaux 1, France

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Abstract

The well known NP-hard problem **Partition** is the following. Given a set S of n non-negative integers; partition S into two sets X and Y such that the sum of the elements in X is as close as possible to the sum of the elements in Y (that is, minimize the maximum of the two sums). In this talk we study the following generalization of the partition problem: given an edge-weighted graph G containing two edge-disjoint spanning trees. Find a pair of edge-disjoint spanning trees such that the maximum weight of these two trees is as small as possible. In the case when G is precisely the union of two trees this problem may be seen as a generalization of the partition problem in which we have added a graph structure to the numbers (through the edges) and the extra restriction that only sets X and Y which correspond to trees in G are valid partitions. We show how to combine a reduction heuristic (based on an algorithm for weighted matroid partition) for the problem with local search methods to obtain high quality solutions for this problem and compare these to solutions obtained by an approximation algorithm which uses the same reduction heuristic. Finally, we discuss possible implications of these findings.