An inequality for Tutte polynomials

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Abstract

Let G be a graph without loops or bridges and a, b be positive real numbers with $b \geq a(a+2)$. We show that the Tutte polynomial of G satisfies the inequality $T_G(b,0)T_G(0,b) \geq T_G(a,a)$. Our result was inspired by a conjecture of Merino and Welsh that $T_G(1,1) \leq \max\{T_G(2,0), T_G(0,2)\}$. Note that $T_G(1,1)$ is the number of spanning trees of G, $T_G(2,0)$ is the number of acyclic orientations of G, and $T_G(0,2)$ is the number of strongly connected orientations of G.