Tashkinov trees and edge-colourings of multigraphs

Diego Scheide
TU Ilmenau

For the chromatic index $\chi'(G)$ of a (multi)graph G, there are two trivial lower bounds, namely the maximum degree $\Delta(G)$ and the maximum density

$$w(G) = \max_{H \subseteq G, |V(H)| \geq 2} \lceil |E(H)| / \lfloor |V(H)|/2 \rfloor \rceil .$$

A famous conjecture due to Goldberg and Seymour says that every graph G satisfies $\chi'(G) \leq \max\{\Delta(G)+1, w(G)\}$. This means that $\chi'(G) = w(G)$ for every graph G with $\chi'(G) \geq \Delta(G)+2$. The considered class of graphs $\mathcal{J}$ can be subdivided into an ascending sequence of classes $(\mathcal{J}_m)_{m \geq 3}$. For $m \leq 15$ the conjecture could already be proved leading to the main result that

$$\chi'(G) \leq \max \left\{ \left\lfloor \frac{15}{14}\Delta(G) + \frac{12}{14} \right\rfloor , w(G) \right\}$$

for every graph G. The proof is mainly based on recolouring techniques on structures called Tashkinov trees. An overview of this general colouring concept and some other related results will be given.