Double-Critical and Contraction-Critical Graphs

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Abstract

In this talk, mostly based on notes by Bjarne Toft, we present new results on double-critical graphs and on graphs which are both double-critical and contraction-critical. A connected graph G is said to be double-critical if for any edge $xy \in E(G)$, $\chi(G - \{x, y\}) \leq \chi(G) - 2$, and contraction-critical if for any proper minor H of G, $\chi(H) < \chi(G)$. Lovász conjectured [*Theory of Graphs: Proc. Colloquium, Hungary*, 1966] that the only double-critical k-chromatic graph ($k \geq 1$) is the complete k-graph. This conjecture has been proved true for $k \leq 5$ [Stiebitz, *Discrete Math.* **64**, 91–93, 1987], but remains open for $k \geq 6$. We prove, without using the Four Colour Theorem, that any double-critical 6-chromatic graph contains K_6 as a minor, and that any double-critical 7-chromatic graph contains K_7^-. Hadwiger's Conjecture [1943] states that the only contraction-critical k-chromatic ($k \geq 1$) graph is the complete k-graph; it remains open for $k \geq 7$. Lovász' Conjecture and Hadwiger's Conjecture have the same conclusions, and so, by combining the assumption of Lovász' Conjecture with the assumption of Hadwiger's Conjecture we obtain a conjecture with a stronger assumption. For this combined-Hadwiger-Lovász Conjecture we prove that the only double-critical contraction-critical 7-chromatic graph is the complete 7-graph.