Approximating Range Assignment Problems

Bernhard Fuchs, TU Braunschweig
bfuchs@ibr.cs.tu-bs.de

Applications in wireless network design motivate the study of the following problem setting. We are given a set of transmitter/receiver stations V with their pairwise distances $w : V \times V \to \mathbb{R}_+$ and a network property Π. The task is to assign transmission ranges $r : V \to \mathbb{R}_+$ to each station such that the (directed or undirected) network *induced* by r satisfies property Π with least possible total energy consumption $\sum_{v \in V} r(v)$. An arc (v, u) is induced by r if $r(v) \geq w(u, v)$, and an edge $\{v, u\}$ is induced by r if $\min\{r(u), r(v)\} \geq w(u, v)$. In this talk, we address three choices of Π:

- The induced undirected network shall be *connected*. (C)
- The undirected network shall be *strongly connected*. (SC)
- The undirected network shall contain an arborescence with root $s \in V$, i.e., s *broadcasts* to all other stations. (B)

Consider the following refomulation of (C): Find a spanning tree $T \subseteq V \times V$ minimizing

$$\sum_{v \in V} \max_{\{v,w\} \in T} w(v, w), \tag{1}$$

and compare it to the 'classic' minimum spanning tree problem where we minimize

$$\frac{1}{2} \sum_{v \in V} \sum_{\{v,w\} \in T} w(v, w). \tag{2}$$

Disregarding the constant factor, comparing (1) to (2) illustrates the step from a 'classic' (wired) network design problems to a (wireless) range assignment problem. We survey recent hardness results on these problems in special settings, e.g. the geometric settings where these problems originate from. We also survey known approximation algorithms as well as giving new results for two greedy strategies that are analogous to Prim's and Kruskal's minimum spanning tree algorithms. The talk closes with several open problems in this area.