Colouring and distinguishing edges by total labellings

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A total k-labelling of a graph $G = (V, E)$ is a function $f : V \cup E \to \{1, 2, \dots, k\}$. The weight of an edge uv is $w(uv) = f(u)+f(uv)+f(v)$. We investigate edge-distinguishing total k-labellings, where all edge weights must be different, and edge-colouring total k-labellings, where the edge weights of incident edges must be different, i.e. they determine a proper edge colouring of G. In both cases we try to minimize k.

Let G be a graph with m edges and maximum degree Δ. In the case of edge-distinguishing total labellings, our main result is that the natural lower bound

$$k \geq \left\lceil \max\left\{\frac{m+2}{3}, \frac{\Delta+1}{2}\right\}\right\rceil$$

is tight for all graphs with $m \geq 111000\Delta$. Ivančo and Jendrol' conjecture that the bound is tight for all $G \neq K_5$.

In the case of edge-colouring total labellings the natural lower bound is $k \geq \lceil \frac{\Delta+1}{2} \rceil$. This lower bound cannot be tight in general, but we are not aware of any graph, where k must exceed the lower bound by more than one. Our main result here is an upper bound of $k \leq \frac{\Delta}{2} + \mathcal{O}(\sqrt{\Delta \log \Delta})$. In both cases we employ a mixture of graph theoretic and probabilistic methods.

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