Linkedness and ordered cycles in directed graphs

Deryk Osthus

October 22, 2007

Abstract

Given a digraph D, let $\delta^0(D) := \min\{\delta^+(D), \delta^-(D)\}$ be the minimum semi-degree of D. We show that every sufficiently large digraph Dwith $\delta^0(D) \ge n/2 + \ell - 1$ is ℓ -linked. The bound on the minimum semidegree is best possible and confirms a conjecture of Manoussakis from 1990. We also determine the smallest minimum semi-degree which ensures that a sufficiently large digraph D is k-ordered, i.e. that for every sequence s_1, \ldots, s_k of distinct vertices of D there is a directed cycle which encounters s_1, \ldots, s_k in this order. In fact, we prove that the latter semidegree even ensures D to be k-ordered Hamiltonian. This is joint work with Daniela Kühn and (for the Hamiltonicity result) Andrew Young.