Vizing's Colouring Algorithm and the Fan Number

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If we colour the vertices of a graph G by a greedy algorithm, then, for the number $A(G)$ of colours used by the algorithm, we have $A(G) \leq \Delta(G) + 1$. In general, though, this upper bound is rather generous. Indeed, if we choose a smallest last ordering of the vertices of G, we obtain $A(G) \leq \mathrm{col}(G) + 1$, where $\mathrm{col}(G)$ denotes the *colouring number* of G.

If we colour the edges of a multigraph G by Vizing's fan algorithm, then, for the number $A'(G)$ of colours used by the algorithm, we have $A'(G) \leq \Delta(G) + \mu(G)$, where $\mu(G)$ denotes the *maximum multiplicity*. We introduce a new graph parameter $\sigma(G)$ - the so called *fan number* of G. The fan number behaves similar to the colouring number. In particular, by choosing a suitable edge ordering to start with, we obtain $A'(G) \leq \max\{\Delta(G), \sigma(G)\} =: \tau(G)$. Hence τ is an efficiently realizable upper bound for the chromatic index χ'. Furthermore, τ is a lower bound for several well known upper bounds for χ', including the bounds by Vizing, Shannon and Ore.