COMPUTING THE TUTTE POLYNOMIAL IN VERTEX-EXPONENTIAL TIME*

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ABSTRACT. The deletion–contraction algorithm is perhaps the most popular method for computing a host of fundamental graph invariants such as the chromatic, flow, and reliability polynomials in graph theory, the Jones polynomial of an alternating link in knot theory, and the partition functions of the models of Ising, Potts, and Furtuin–Kasteleyn in statistical physics. Prior to this work, deletion–contraction was also the fastest known general-purpose algorithm for these invariants, running in time roughly proportional to the number of spanning trees in the input graph. Here, we provide a substantially faster algorithm that computes the Tutte polynomial—and hence, all the aforementioned invariants and more—of an arbitrary n-vertex graph in time and space $2^n n^{O(1)}$. The algorithm is based on a new recurrence formula that alternates between partitioning an induced subgraph into components and a subtraction step to solve the connected case. For bounded-degree graphs the algorithm runs in time $(2-\epsilon)^n$ for an $\epsilon > 0$ that depends only on the degree bound. An implementation of the algorithm outperforms deletion–contraction also in practice.

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