

Complete minors in large 6-connected graphs

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In general, the structure of graphs with no K_6 minor appears to be complicated. However, Jørgensen has conjectured that every six connected graph G that does not contain K_6 as a minor must contain a vertex v such that $G - v$ is a planar graph. One immediate implication of the conjecture would be to give an alternate proof of the $k = 6$ case of Hadwiger's conjecture. We prove that the conjecture is true with the additional assumption that the graph is very large.

Theorem 1 *There exists a positive integer N such that every 6-connected graph G with $|V(G)| \geq N$ either contains K_6 as a minor or there exists a vertex v such that $G - v$ is planar.*

We give an outline of the proof of the theorem and the techniques used. We conclude with a brief discussion of possible extensions of Jørgensen's conjecture.