Total domination in 2-connected graphs and transversals in hypergraphs

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Abstract

A set S of vertices in a graph G is a total dominating set of G if every vertex of G is adjacent to some vertex in S. The minimum cardinality of a total dominating set of G is the total domination number $\gamma_t(G)$ of G. It is known [J. Graph Theory 35 (2000), 21–45] that if G is a connected graph of order $n > 10$ with minimum degree at least two, then $\gamma_t(G) \leq 4n/7$ and the (infinite family of) graphs of large order that achieve equality in this bound are characterised.

We improve this upper bound of $4n/7$ for 2-connected graphs, as well as for connected graphs with no induced 6-cycle. We prove that if G is a 2-connected graph of order $n > 18$, then $\gamma_t(G) \leq 6n/11$. Our proof is an interplay between graph theory and transversals in hypergraphs. We also prove that if G is a connected graph of order $n > 18$ with minimum degree at least two and no induced 6-cycle, then $\gamma_t(G) \leq 6n/11$. Both bounds are sharp.

In this talk we will furthermore mention a few open problems and results on total domination and transversals in hypergraphs.