Antistrong digraphs

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An *antidirected trail* in a digraph is a trail (a walk with no arc repeated) in which the arcs alternate between forward and backward arcs. An *antidirected path* is an antidirected trail where no vertex is repeated. We show that it is *NP*-complete to decide whether two vertices x, y in a digraph are connected by an antidirected path, while one can decide in linear time whether they are connected by an antidirected trail. Indeed this can be done easily by looking for an (x, y)-path in the bipartite representation of the input digraph.
Motivated by this observation, we say that a digraph is *antistrong* if its bipartite representation is connected. We study natural algorithmic questions on antistrong digraphs. Some results derive easily from the definition (eg. checking k-arc-antistrongness, or computing the minimum number of arcs to add to be antistrong....), other are more difficult to establish. In particular, we use results from matroid theory to characterize graphs which admit an antistrong orientation and give a polynomial time algorithm for constructing such an orientation when it exists.

In this talk, I will present these notions and results and also give some problems and conjectures related to this area.

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