BOXICITY AND TOPOLOGICAL INVARIANTS

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Abstract. The boxicity of a graph $G = (V, E)$ is the smallest integer k for which there exist k interval graphs $G_i = (V, E_i)$, $1 \leq i \leq k$, such that $E = E_1 \cap \cdots \cap E_k$. Equivalently, the boxicity of G is the smallest integer $d \geq 1$ such that G is the intersection graph of a collection of boxes in $\mathbb{R}^d$ (a box in $\mathbb{R}^d$ the cartesian product of d closed intervals of the real line).

In the first part of the talk, I will prove that every graph on m edges has boxicity $O(\sqrt{m \log m})$, which is asymptotically best possible. I will use this result to study the connection between the boxicity of graphs and their Colin de Verdière invariant, which share many similarities. Known results concerning the two parameters suggest that for any graph G, the boxicity of G is at most the Colin de Verdière invariant of G, denoted by $\mu(G)$. We will observe that every graph G has boxicity $O(\mu(G)^4(\log \mu(G))^2)$, while there are graphs G with boxicity $\Omega(\mu(G)\sqrt{\log \mu(G)})$.

In the second part of the talk, I will focus on graphs embeddable on a surface of Euler genus g. I will prove that these graphs have boxicity $O(\sqrt{g} \log g)$, while some of these graphs have boxicity $\Omega(\sqrt{g \log g})$. The proof of the upper bound is based on recent results on acyclic coloring of graphs on surfaces. These results directly imply a nearly optimal bound on the dimension of the adjacency poset of graphs on surfaces.

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