Unique Completability of Partially Filled Low Rank Positive Semidefinite Matrices

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Abstract: We consider the problems of completing a low-rank positive semidefinite square matrix M or a low-rank rectangular matrix N from a given subset of their entries. Following the approach initiated by Singer and Cucuringu we study the local and global uniqueness of such completions by analysing the structure of the graphs determined by the positions of the known entries of M or N.

We present combinatorial characterizations of local and global (unique) completability for special families of graphs. We characterize local and global completability in all dimensions for cluster graphs, i.e. graphs which can be obtained from disjoint complete graphs by adding a set of independent edges. These results correspond to theorems for body-bar frameworks in rigidity theory. We provide a characterization of two-dimensional local completability of planar bipartite graphs, which leads to a characterization of two-dimensional local completability in the rectangular matrix model when the underlying bipartite graph is planar. We also give bounds on the number of known entries per row, or in total, which guarantee local or global completability. These results are based on new observations that certain graph operations preserve local or global completability, as well as on a further connection between rigidity and completability.

We also show that a rank condition on the completability stress matrix of a graph is a sufficient condition for global completability. This verifies a conjecture of Singer and Cucuringu.