Mixed Moore Graphs

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A Moore graph of diameter D is a graph with the property that for any two vertices x, y there is a unique path of length at most D from x to y.

For a Mixed Moore graph of diameter 2, there are numbers t and z so that every vertex is incident with t undirected edges and has z out-neighbors and z in-neighbours.

Bosák (1979) proved that $t = \frac{c^2+3}{4}$ where c divides $(4z-3)(4z+5)$. Bosák also constructed a mixed Moore graph with 18 vertices and $t = 3, z = 1$. It is also known (Gimbert 2001) that there is a unique mixed Moore graph with $t = 1$ and any z: the linedigraph of the complete digraph, $L(K_{z+2})$.

I was able to find a new mixed Moore graph with 108 vertices and $t = 3, z = 7$. This graph was found in a computer search for Cayley graphs with these properties.

Bosák's graph is also a Cayley graph and we prove from the characterization of sharply 2-transitive groups (Zassenhaus 1936) that $L(K_{z+2})$ is Cayley graph if and only if $z + 2$ is a prime-power.

Finally, we consider attempts to prove non-existence of mixed Moore graphs for some (small) feasible values of t and z.