

# Unique colorability and clique minors

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July 30, 2015

## Abstract

For a graph  $G$ , let  $h(G)$  denote the largest  $k$  such that  $G$  has  $k$  pairwise disjoint pairwise adjacent connected nonempty subgraphs, and let  $s(G)$  denote the largest  $k$  such that  $G$  has  $k$  pairwise disjoint pairwise adjacent connected subgraphs of size 1 or 2. HADWIGER's conjecture states that  $h(G) \leq \chi(G)$ , where  $\chi(G)$  is the chromatic number. SEYMOUR conjectured  $s(G) \geq |V(G)|/2$  for all graphs without antitriangles, i. e. three pairwise nonadjacent vertices. Here we concentrate on graphs with exactly one  $\chi(G)$ -coloring. We prove generalizations of (i) if  $\chi(G) \leq 6$  and  $G$  has exactly one  $\chi(G)$ -coloring then  $h(G) \geq \chi(G)$ , where the proof does *not* use the four-color-theorem, and (ii) if  $G$  has no antitriangle and  $G$  has exactly one  $\chi(G)$ -coloring then  $s(G) \geq |V(G)|/2$ .

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