Critical vertices in k-connected digraphs

Abstract

It is proved that every non-complete, finite digraph of connectivity number k has a fragment F containing at most k critical vertices. The following result is a direct consequence: Every k-connected, finite digraph D of minimum outdegree and minimum indegree at least $2k + m - 1$ for positive integers k, m has a subdigraph H of minimum outdegree or minimum indegree at least $m - 1$ such that $D - x$ is k-connected for all vertices x of H. For $m = 1$, this implies immediately the existence of a vertex of indegree or outdegree less than $2k$ in a k-critical finite digraph, which was proved in my paper [Ecken von kleinem Grad in kritisch n-fach zusammenhngenden Digraphen, JCT(B) 53 (1991), 260-272)].