Decomposing highly edge-connected graphs into trees of small diameter

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Abstract

The Tree Decomposition Conjecture by Bárat and Thomassen states that for every tree T there exists a natural number k(T) such that the following holds: If G is a k(T)-edge-connected simple graph with size divisible by the size of T, then G can be edge-decomposed into subgraphs isomorphic to T.

The results on modulo k-orientations by Thomassen show that the Tree Decomposition Conjecture holds for all stars. Recently Bensmail, Harutyunyan and Thomassé proved the conjecture for all paths. Apart from trees that are paths or stars, it has only been verified for a certain class of bistars by Thomassen.

We sketch the proof of a weaker version of the Tree Decomposition Conjecture, where we require the subgraphs in the decomposition to be isomorphic to graphs that can be obtained from T by vertexidentifications. This implies the Tree Decomposition Conjecture under the additional constraint that the girth of G is greater than the diameter of T. We also indicate how this result implies the original Tree Decomposition Conjecture for all trees of diameter at most 4, as well as for some trees of diameter 5.