New results for variants of the Merino-Welsh conjecture

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We present new results for the Merino-Welsh conjecture and its two variants, namely the multiplicative version and the convex version.

Let $a(G)$, $c(G)$, $t(G)$ be the numbers of acyclic orientations, totally cyclic orientations and spanning trees of a graph G respectively. The Merino-Welsh conjecture claims that if G is a 2-connected loopless graph, then $t(G) \leq \max\{a(G), c(G)\}$. The multiplicative version is to show $t(G)^2 \leq a(G)c(G)$, and the convex version claims that the Tutte polynomial is convex on the line segment between the points $(2, 0)$ and $(0, 2)$, on which all the numbers $a(G), c(G)$ and $t(G)$ lie.

We prove that, if G has n vertices and m edges, then
$t(G) \leq a(G)$ if $m \leq 1.29(n - 1)$, and
$t(G) \leq c(G)$ if G is 3-edge-connected and $m \geq 3.58(n - 1)$,
which improve Thomassen's result.

Also, inspired by Noble and Royle's proof for series-parallel graphs, we prove the multiplicative version for graphs of pathwidth at most 3 using a computer search. The convex version holds for minimally 2-edge-connected graphs.