Optimal Colorings with Rainbow Paths

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Let G be a connected graph of chromatic number k. For a k-coloring f of G, a full f-rainbow path is a path of order k in G whose vertices are all colored differently by f.

We show that G has a k-coloring f such that every vertex of G lies on a full f-rainbow path, which provides a positive answer to a question posed by Lin (Simple proofs of results on paths representing all colors in proper vertex-colorings, Graphs Combin. 23 (2007) 201-203). Furthermore, we show that G has a k-coloring f such that, for every vertex u of G, some full f-rainbow path begins at u provided that G has a cycle of length k, which solves a problem posed by Bessy and Bousquet (Colorful paths for 3-chromatic graphs, arXiv 1503.00965v1) and verifies a special case of a conjecture of Akbari et al. (Colorful paths in vertex coloring of graphs, preprint). Finally, we establish some more results on the existence of optimal colorings with (directed) full rainbow paths.

The presented results are joint work with Oliver Bendele (Ulm University).