From finite to infinite graphs

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Abstract

Nash-Williams proved in 1960 that
(1) a graph has an edge-decomposition into cycles if and only if it has no odd cut.
The proof is trivial in the finite case, an easy exercise in the countable case, and complicated in the uncountable case.

Laviolette used in 2005 Nash-Williams theorem to prove that
(2) every k-edge-connected graph has an edge-decomposition into countable k-edge-connected graphs.

Laviolette's result immediately implies that
(3) every bridgeless graph has a collection of cycles such that every edge is in at least one and at most countably many cycles.

I shall indicate a (relatively) simple proof of (3) and explain how (3) implies Nash-Williams' theorem. Thus (1),(2),(3) are equivalent in the sense that one relatively easily implies the others. Furthermore, I can replace "at most countably many" in (3) by "at most 7". If 7 can be replaced by 2 the strong cycle-double-cover-conjecture would follow.

Finally, I shall indicate a proof of the result that every graph of uncountable chromatic number contains a subgraph of infinite edge-connectivity.