Deterministic Edge Connectivity in Near-Linear Time

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Abstract

We present a deterministic algorithm that computes the edge-connectivity of a graph in near-linear time. This is for a simple undirected unweighted graph G with n vertices and m edges. This is the first $o(mn)$ time deterministic algorithm for the problem. Our algorithm is easily extended to find a concrete minimum edge-cut. In fact, we can construct the classic cactus representation of all minimum cuts in near-linear time.

The previous fastest deterministic algorithm by Gabow from STOC'91 took $\tilde{O}(m + \lambda^2 n)$, where λ is the edge connectivity, but λ can be as big as $n - 1$.

At STOC'96 Karger presented a randomized near linear time Monte Carlo algorithm for the minimum cut problem. As he points out, there is no better way of certifying the minimality of the returned cut than to use Gabow's slower deterministic algorithm and compare sizes.

Our main technical contribution is a near-linear time algorithm that contracts vertex sets of a simple input graph G with minimum degree δ, producing a multigraph $\overline{G}$ with $\tilde{O}(m/\delta)$ edges which preserves all minimum cuts of G with at least two vertices on each side.

In our deterministic near-linear time algorithm, we will decompose the problem via low-conductance cuts found using PageRank a la Brin and Page (1998), as analyzed by Andersson, Chung, and Lang at FOCS'06. Normally such algorithms for low-conductance cuts are randomized Monte Carlo algorithms, because they rely on guessing a good start vertex. However, in our case, we have so much structure that no guessing is needed.

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