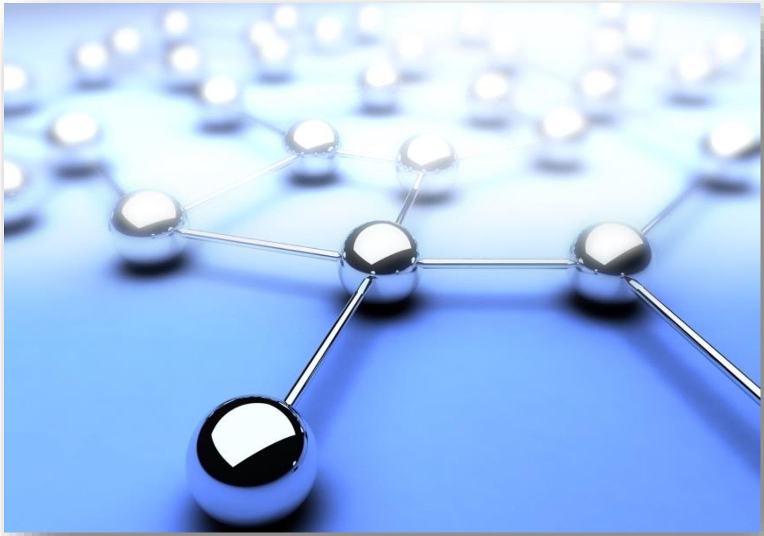
The Price of Leasing Online
TOLA 2014



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July 7, 2014

Joint work with:

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Outline

- Parking Permit Problem
- Infrastructure Leasing Problems
- Set Cover Leasing
- The Price of Leasing Online

Parking Permit Problem

sunny day

walk

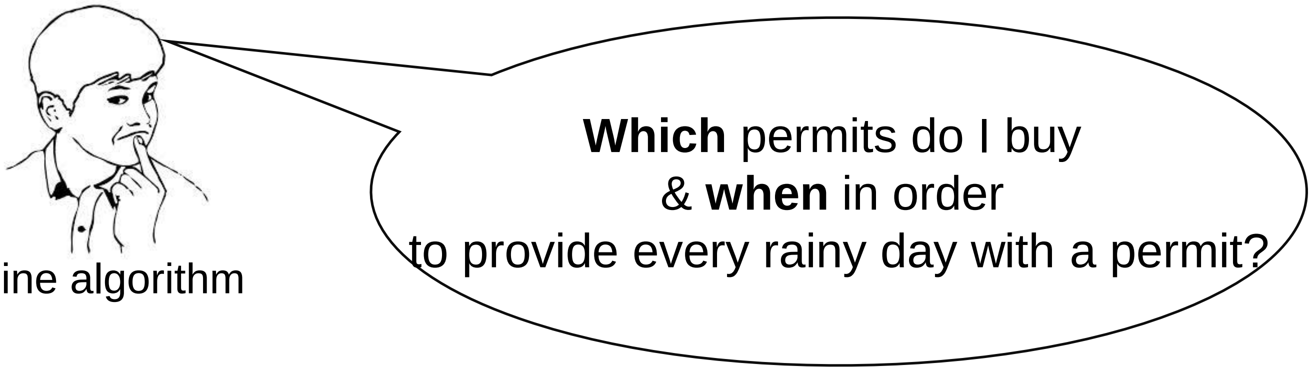
rainy day

drive

[Meyerson - FOCS 2003]

Parking Permit Problem

- adversary gives **sunny** or **rainy** on each day
- ***K*** permit *lease* types (Ex. daily, weekly, monthly, yearly)
- yearly permit is the **most expensive but cheapest per day**



Online algorithm

quality of online algorithm → competitive factor α

- **adversary** reveals in each step part of overall input
- $\alpha = \max \frac{\textbf{cost of Online algorithm}}{\textbf{cost of Optimal Offline algorithm}}$ over all input instances

Optimal Offline
knows
the future in advance

[Meyerson - FOCS 2003]

Parking Permit Problem

Lower bounds	Upper bounds
$\Omega(K)$ deterministic	$O(K)$ deterministic
$\Omega(\log K)$ randomized	$O(\log K)$ randomized

Deterministic algorithm

For each rainy day, buy a 1-day permit, until there is some $(k \in K)$-interval where the **optimum offline solution** for the sequence of days seen so far, would buy a k-day permit. In this case, also buy a k-day permit.

Randomized algorithm

Compute an $O(\log K)$-competitive fractional solution and then convert it into a randomized integer solution which maintains the $O(\log K)$-competitive factor.

[Meyerson - FOCS 2003]

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Infrastructure Leasing Problems

Leasing Company

Client 1

Client 2

Client 3

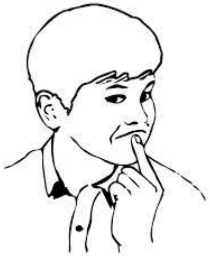
.
.
.

Provider 1

Provider 2

Provider 3

.
.
.



Long lease or short lease ….. ?

Infrastructure Leasing Problems

- Almost any online infrastructure problem can be considered with a *leasing* aspect….

- Anthony & Gupta generalized the Parking Permit Problem
  - Facility Leasing
  - Steiner Tree Leasing
  - Set Cover Leasing

  & gave **offline** algorithms to the problems…

[Anthony et al.- IPCO 2007]

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Set Cover Leasing

Online Set cover

- $U = \{e_1, e_2, \ldots, e_n\}$
- family $F = \{S_1, S_2, \ldots, S_m\}$ of subsets of U and a cost associated with each subset
- an element $e \in U$ arrives

-- choose sets from F to cover each arriving element $e \in U$ & minimize cost of sets --

Set Cover Leasing

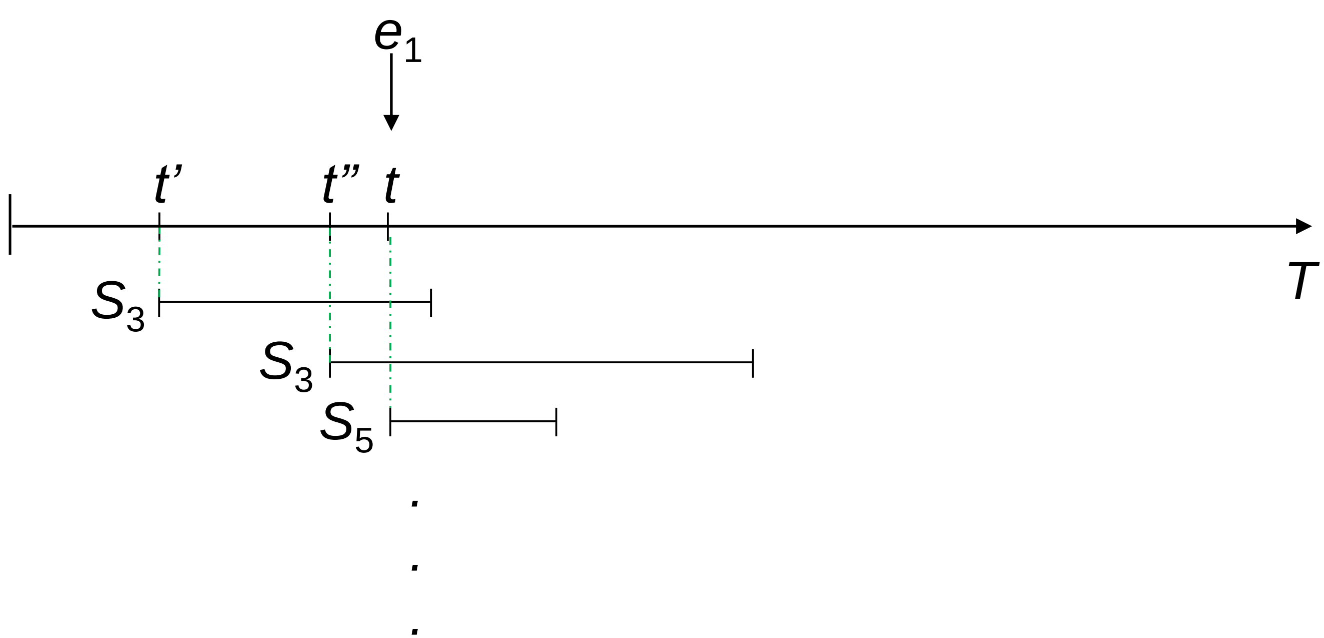
- $U = \{e_1, e_2, \ldots, e_n\}$
- family $F = \{S_1, S_2, \ldots, S_m\}$ of subsets of U. Each set in F can be leased for K different periods of time such that leasing a set S for a period k :
  - incurs a cost c_{kS}
  - allows S to cover its elements for the next l_k time steps
- an element $e \in U$ arrives

-- lease sets from F to cover each arriving element $e \in U$ & minimize cost of sets --

generalizes **Online Set Cover** (K = 1)
- one infinite lease -

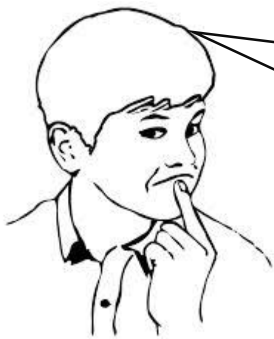
Set Cover Leasing

- e_1 arrives at time t
- $e_1 \in \{S_3, S_5, S_8\}$



Set Cover Leasing

Ex. servers/clients in a computer network



network manager

a client arrives in each step

Which servers shall I install in order to
serve each arriving client
while minimizing cost ??

Once a server is installed, it serves its clients **forever** without additional costs…
[Online set cover]

If servers are **leased** instead & can serve their clients only during the time they are leased…
[Set cover leasing]

Set Cover Leasing

Lower bounds

-none

-related problems

-Online Metric Facility Location: $\Omega(\frac{\log n}{\log\log n})$

[ICALP 2003]

-Online Set Cover: $\Omega(\frac{log\ n \log m}{log\ log\ n+\log\log m})$

[STOC 2003]

Upper bounds

-Online Metric Facility Leasing: $\mathrm{O}(K\ log\ n)$

[IPCO 2008]

-An algorithm for Online Facility Leasing: $\mathrm{O}(l\_max\ log\ l\_max)$

[SIROCCO 12]

-Randomized Online Algorithms for Set Cover Leasing Problems: $\mathrm{O}(\log(mK)\ log\ n)$

[submitted to WAOA]

Set Cover Leasing

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University of Paderborn
Algorithms and Complexity

Algorithm {Set Cover Leasing}

Maintain a fraction f_{Skt} for each set (S, k, t)

- set to 0 initially
- non-decreasing throughout algorithm

Maintain for each set (S, k, t)

- $2\lceil \log(n+1) \rceil$ independent random variables $X_{(Skt)(q)}$ in $[0,1]$
- Let $\mu_{Skt} = \min X_{(Skt)(q)}, 1 \leq q \leq 2\lceil \log(n+1) \rceil$

(*j*, *t*) arrives,

i. (fractional) If $\sum_{(S,k,T) \in Q_j} f_{Skt} < 1$, do the following increment

$$while \ \sum_{(S,k,T) \in Q_j} f_{Skt} < 1;$$

$$f_{Skt} = f_{Skt} \cdot \left(1 + \frac{1}{c_{kS}}\right) + \frac{1}{|Q_j| \cdot c_{kS}}$$

ii. (integer) Lease (S, k, T) ∈ Q_j with $f_{Skt} > \mu_{Skt}$

iii. If (j, t) is not covered by some set in Q_j

Lease the cheapest (S, k, T) ∈ Q_j

- ✓ **Given**: $F = \{S_1, S_2, \ldots, S_m\}$, K leases, $U = \{e_1, e_2, \ldots, e_n\}$
- ✓ **(S, k, T):** $S \in F$, lease k, interval T
- ✓ **(j, t):** $i \in U$, arrives at time t
- ✓ (S, k, T) is a ***candidate*** of (j, t,), if $j \in S$ & $t \in T$
- ✓ Q_j *is the set of candidates of j*

$\boldsymbol{O(\log(dK)\ \log n) - competitive}$

(i) $fractional \leq O(\log(dK)) \cdot Opt$

(ii) $randomized\ integer \leq O(\log n) \cdot fractional$

(iii) $step\ iii\ adds\ an\ expected\ cost\ of\ Opt/n$

Set Cover Leasing

Algorithm {i-cover}

(j, t) arrives.

i. (fractional) If $\sum_{(S,k,T)\in Q_j} f_{Skt} < 1$, do the following increment

$$while\ \sum_{(S,k,T)\in Q_j} f_{Skt} < 1;$$

$$f_{Skt} = f_{Skt} \cdot \left(1 + \frac{1}{c_{kS}}\right) + \frac{1}{|Q_j| \cdot c_{kS}}$$

ii. (integer) Lease (S, k, T) $\in Q_j$ with $f_{Skt} > \mu_{Skt}$

iii. If (j, t) is not covered by some set in Q_j Lease the cheapest (S, k, T) $\in Q_j$

$\boldsymbol{O(\log(dK)\ \log n) - competitive}$

(i) $fractional \leq O(\log(dK)) \cdot Opt$

(ii) $randomized\ integer \leq O(\log n) \cdot fractional$

(iii) $step\ iii\ adds\ an\ expected\ cost\ of\ Opt/n$

Proof: (i)

- an $increment$ adds at most 2 to the $fractional\ cost$

$$\sum_{(S,k,T)\in Q} \frac{c_S \cdot f_{Skt}}{c_S} + \frac{1}{|Q| \cdot c_S} \cdot = \underbrace{\sum_{(S,k,T)\in Q} f_{Skt}}_{<1} + 1 \leq 2$$

- the total number of $increments$ in the algorithm is $O(\log(dK)) \cdot Opt$
  - At any time the algorithm decides to make an $increment$, $\exists\, S_{opt}$ which is a candidate and therefore increases its fraction $f_{S_{opt}kt}$
  - After $O(c_S \cdot \log|Q|)\ increments,\ f_{S_{opt}kt} > 1 \rightarrow \sum_{(S,k,T)\in Q} f_{Skt} > 1$
  - $|Q| \leq d \cdot K$ [Interval Model: Same sets same leases do not coincide]

Parking Permit Problem

Set Cover Leasing

Proof: (ii) $randomized\ integer\ \leq\ O(\log n)\ \cdot fractional$

- Probability to lease a set is $Pr(\ f_{Skt} > \mu_{Skt}\ )$
- $\mu_{Skt} = \min X_{(Skt)(q)}, 1 \leq q\ \leq 2\lceil\log(n+1)\rceil$

Proof: (iii) $step\ iii\ adds\ an\ expected\ cost\ of\ Opt/n$

- [Algorithm leases the cheapest (*S*, *k*, *T*) ∈ Q] $c_S \leq Opt$
- *Probability that an element is not covered* [for a single q] *is at most*

$$\prod_{(Skt)\in Q}(1\ -\ f_{Skt}) \leq\ e^{-\underbrace{\sum_{(S,k,T)\in Q} f_{Skt}}_{\geq 1}} \leq 1/e$$

- *Probability that an element is not covered is at most* $1/n^2$
- $additional\ expected\ cost\ \leq n\ \cdot \frac{1}{n^2} \cdot Opt$

$$\rightarrow O(log\ (dK)\ \ log\, n) - competitive$$

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The Price of Leasing Online

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Algorithms and Complexity

Lower bounds

Online Set Cover: $\Omega(\frac{\log n \log m}{\log\log n + \log\log m})$
+
Parking Permit Problem: $\Omega(K)$
?

Online Facility Location : $\Omega(\frac{\log n}{\log\log n})$
+
Parking Permit Problem: $\Omega(K)$
?

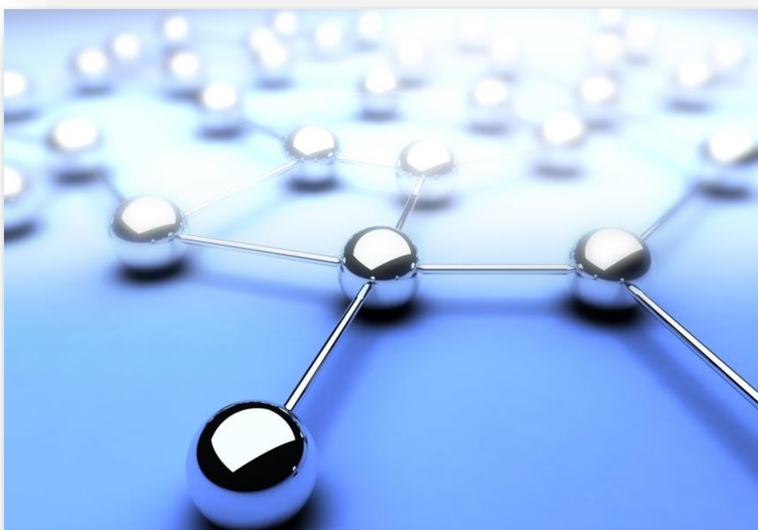
…..

Leasing algorithms so far use techniques from non-leasing algorithms & Parking Permit Problem...

Does leasing impose an inherent difficulty?

What is the price we pay for leasing?

Thank you for your attention!



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