DM582 Exercises - Sheet 11

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This document contains exercises from the course DM582 (spring 2025). Most exercises are from the book *Introduction to Algorithms, 4th edition* by Cormen, Leiserson, Rivest, and Stein (CLRS), the book *Algorithm Design, 1st edition* by J. Kleinberg and E. Tardos (KT), and the book *Discrete Mathematics and its Applications, 8th edition* by K. Rosen.

References to CLRS refer to the book *Introduction to Algorithms, 4th edition* by Cormen, Leiserson, Rivest, and Stein.

References to KT refer to the book *Algorithm Design, 1st edition* by J. Kleinberg and E. Tardos.

References to Rosen refer to the book *Discrete Mathematics and its Applications, 8th edition* by K. Rosen.

References to BG refer to the book *Computer Algorithms: Introduction to Design and Analysis, 3rd edition* by Sara Baase and Allen Van Gelder.

This document will inevitably contain mistakes. If you find some, please report them to your TA so that we can correct them.

Sheet 11

Exercise from course webpage

Exercise

A core problem for an online machine scheduling algorithm is whether to make a level or a skewed schedule. For instance, just considering $m = 2$ and the sequences 1, 1 and 1, 1, 2, we do not know what the best option is for the second job: placing it on the *same* machine as the first job *or not*. (Because this depends on the future: do we receive a third job of size 2 or not?) Try to randomize the decision as we did for the Treasure Hunt problem in the lecture. Assuming you get one of these two sequences, can you get an expected ratio smaller than $\frac{3}{2}$?

Exercise from course webpage

Exercise

A Professor Larsen with extremely poor eyesight is at a long, long wall. He's told that there is a hole somewhere where he can get out, but he won't be able to see it unless he's right at it; and he doesn't know if it's to the left or to the right. The hole is a whole number of steps away from the professor. Devise an algorithm the professor can use to escape. What's the competitive ratio of your algorithm? (Of course, $\textsc{Opt}$ is just the distance from the professor to the hole.)

Exercise from course webpage

Exercise

Show an example where lazy $\textsc{Dc}$ benefits from being lazy and one where it doesn't.

Exercise from course webpage

Exercise

Show that if there exists an infinite family of sequences such that 1) for some fixed constant $b > 0$, we have that $\textsc{Alg}(\sigma) \geq k\textsc{Opt}(\sigma) - b$ for any σ in the family, and 2) for any d, there exists a σ in the family such that

$\textsc{Opt}(\sigma) > d$, then ALG cannot be $(k - \epsilon)$-competitive for any $\epsilon > 0$.

Exercise from course webpage

Exercise

Prove that there exists a minimum weight matching where the server DC moves (when it only moves one) is matched to the server OPT just moved.

Exercise from course webpage

Exercise

Discuss whether or not one should assume that the adversary knows the coin flips of the algorithm - in the light that we want results in our model to say something about the real world.

Exercise from course webpage

Exercise

Consider how one might generalize DC to working on trees. No proofs are expected.