DM582 Exercises - Sheet 10

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This document contains exercises from the course DM582 (spring 2025). Most exercises are from the book *Introduction to Algorithms, 4th edition* by Cormen, Leiserson, Rivest, and Stein (CLRS), the book *Algorithm Design, 1st edition* by J. Kleinberg and E. Tardos (KT), and the book *Discrete Mathematics and its Applications, 8th edition* by K. Rosen.

References to CLRS refer to the book *Introduction to Algorithms, 4th edition* by Cormen, Leiserson, Rivest, and Stein.

References to KT refer to the book *Algorithm Design, 1st edition* by J. Kleinberg and E. Tardos.

References to Rosen refer to the book *Discrete Mathematics and its Applications, 8th edition* by K. Rosen.

This document will inevitably contain mistakes. If you find some, please report them to your TA so that we can correct them.

Sheet 10

CLRS Exercise 32.3-2

Exercise

Draw a state-transition diagram for the string-matching automaton for the pattern *ababbabbabbababbabb* over the alphabet $\Sigma = \{a, b\}$. (The pattern is from Exercise 32.4-1).

CLRS Exercise 32.4-1

Exercise

Compute the prefix π function for the pattern *ababbabbabbababbabb*. Compare with what you got in Exercise 32.3-2.

CLRS Exercise 32.4-3

Exercise

Explain how to determine the occurrences of pattern P in the text T by examining the π function for the string PT (the string of length $m + n$ that is the concatenation of P and T).

CLRS Exercise 32.4-5

Exercise

Use a potential function to show that the running time of `KMP-MATCHER` is $\Theta(n)$. Think of the whole while-loop as an operation, `update_q`. Prove that operation `update_q` is amortized constant time. (*Algorithm shown below*).

```
KMP-MATCHER(T, P, n, m)
1  π = COMPUTE-PREFIX-FUNCTION(P, m)
2  q = 0                          // number of characters matched
3  for i = 1 to n                 // scan the text from left to right
4      while q > 0 and P[q + 1] ≠ T[i]
5          q = π[q]               // next character does not match
6      if P[q + 1] == T[i]
7          q = q + 1              // next character matches
8      if q == m                  // is all of P matched?
9          print "Pattern occurs with shift" i − m
10         q = π[q]               // look for the next match
```

```
COMPUTE-PREFIX-FUNCTION(P, m)
let π[1 : m] be a new array
π[1] = 0
k = 0
for q = 2 to m
    while k > 0 and P[k + 1] ≠ P[q]
        k = π[k]
    if P[k + 1] == P[q]
        k = k + 1
    π[q] = k
return π
```

CLRS Exercise 32.4-6

Exercise

Show how to improve `KMP-MATCHER` by replacing the occurrence of π in line 5 by (but not line 10) by π', where π' is defined recursively for $q = 1, 2, \ldots, m-1$ by the equation

$$\pi'[q] = \begin{cases} 0 & \text{if } \pi[q] = 0, \\ \pi'[\pi[q]] & \text{if } \pi[q] \neq 0 \text{ and } P[\pi[q]+1] = P[q+1], \\ \pi[q] & \text{if } \pi[q] \neq 0 \text{ and } P[\pi[q]+1] \neq P[q+1]. \end{cases}$$

Explain why the modified algorithm is correct, and explain in what sense this change constitutes an improvement.

CLRS Exercise 32.4-7

Exercise

Give a linear-time algorithm to determine whether a text T is a cyclic rotation of another string T'. For example, `braze` and `zebra` are cyclic rotations of each other.

CLRS Exercise 32.4-8

Exercise

Give an $O(m|\Sigma|)$-time algorithm for computing the transition function δ for the string-matching automaton corresponding to a given pattern P. (Hint: Prove that $\delta(q, a) = \delta(\pi[q], a)$ if $q = m$ or $P[q + 1] \neq a$.)

Note: The hint follows from our discussions at the lecture, where we proved that $\delta(q, a)$ could be computed by following the chain defined by the π function. And the hint just says that we end up the same place if we take just one step along the chain and then continues recursively from there. Thus, just assume that the hint is true and now solve the exercise.