DM582 Exercises - Sheet 12

Mads Anker Nielsen Tobias Samsøe Sørensen

April 22, 2025

This document contains exercises from the course DM582 (spring 2025). Most exercises are from the book *Introduction to Algorithms, 4th edition* by Cormen, Leiserson, Rivest, and Stein (CLRS), the book *Algorithm Design, 1st edition* by J. Kleinberg and E. Tardos (KT), and the book *Discrete Mathematics and its Applications, 8th edition* by K. Rosen.

References to CLRS refer to the book *Introduction to Algorithms, 4th edition* by Cormen, Leiserson, Rivest, and Stein.

References to KT refer to the book *Algorithm Design, 1st edition* by J. Kleinberg and E. Tardos.

References to Rosen refer to the book *Discrete Mathematics and its Applications, 8th edition* by K. Rosen.

References to BG refer to the book *Computer Algorithms: Introduction to Design and Analysis, 3rd edition* by Sara Baase and Allen Van Gelder.

This document will inevitably contain mistakes. If you find some, please report them to your TA so that we can correct them.

Sheet 12

CLRS Exercise 9.3-1

Exercise

In the algorithm `SELECT`, the input elements are divided into groups of 5. Show that the algorithm works in linear time if the input elements are divided into groups of 7 instead of 5.

CLRS Exercise 9.3-3

Exercise

Show how to use `SELECT` as a subroutine to make quicksort run in $O(n \lg n)$ time in the worst case, assuming that all elements are distinct.

CLRS Exercise 9.3-6

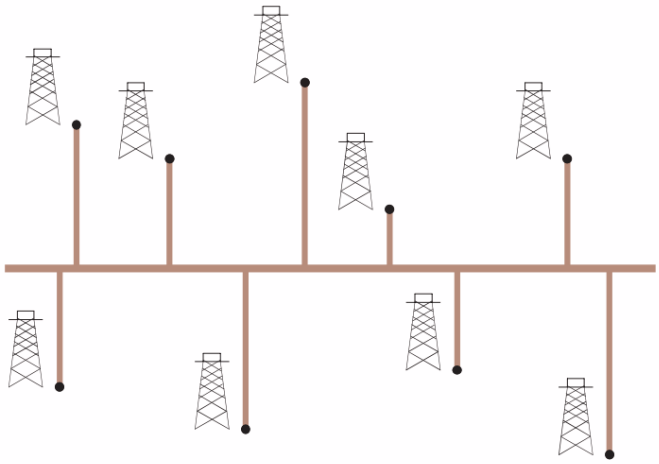
Exercise

You have a "black-box" worst-case linear-time median subroutine. Give a simple, linear-time algorithm that solves the selection problem for an arbitrary order statistic.

CLRS Exercise 9.3-7

Exercise

Professor Olay is consulting for an oil company, which is planning a large pipeline running east to west through an oil field of n wells. The company wants to connect a spur pipeline from each well directly to the main pipeline along a shortest route (either north or south), as shown in Figure 9.4. Given the x- and y-coordinates of the wells, how should the professor pick an optimal location of the main pipeline to minimize the total length of the spurs? Show how to determine an optimal location in linear time. Figure 9.4 shown below.



BG Exercise 5.20

Exercise

Consider the problem of determining if a bit string of length n contains two consecutive zeroes. The basic operation is to examine a position in the string to see if it is a 0 or a 1. For each $n = 2, 3, 4, 5$ either give an adversary strategy to force any algorithm to examine every bit, or give an algorithm that solves the problem by examining fewer than n bits.

BG Exercise 5.21

Exercise

Suppose you have a computer with a small memory and you are given a sequence of keys in an external file (on a disk or tape). Keys can be read into memory for processing, but no key can be read more than once.

a. What is the minimum number of storage cells needed for keys in memory to find the largest key in the file? Justify your answer.

b. What is the minimum number of cells needed for keys in memory to find the median? Justify your answer.

CLRS Exercise 9.3-5

Exercise

Show how to determine the median of a 5-element set using only 6 comparisons.