DM582 Exercises - Sheet 3

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This document contains exercises from the course DM582 (spring 2025). Most exercises are from the book *Introduction to Algorithms, 4th edition* by Cormen, Leiserson, Rivest, and Stein (CLRS), the book *Algorithm Design, 1st edition* by J. Kleinberg and E. Tardos (KT), and the book *Discrete Mathematics and its Applications, 8th edition* by K. Rosen.

References to CLRS refer to the book *Introduction to Algorithms, 4th edition* by Cormen, Leiserson, Rivest, and Stein.

References to KT refer to the book *Algorithm Design, 1st edition* by J. Kleinberg and E. Tardos.

References to Rosen refer to the book *Discrete Mathematics and its Applications, 8th edition* by K. Rosen.

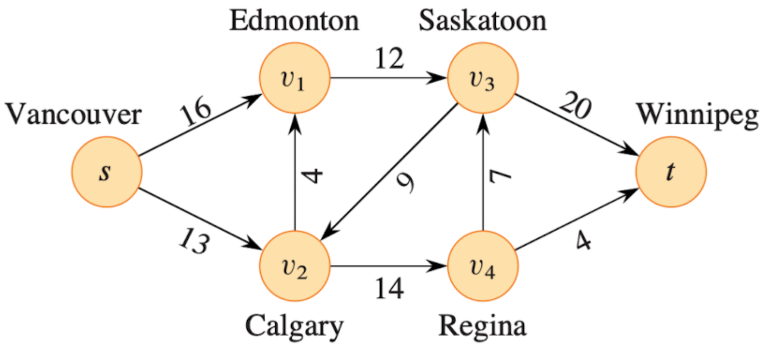
This document will inevitably contain mistakes. If you find some, please report them to your TA so that we can correct them.

Sheet 2

CLRS, 24.2-3

Exercise

Show the execution of the Edmonds-Karp algorithm on the flow network of Figure 24.1(a).



CLRS, 24.2-7

Exercise

Prove lemma 24.2.

Lemma 24.2. Let $G = (V, E)$ be a flow network, let f be a flow in G, and let p be an augmenting path in G_f. Define a function $f_p : V \times V \to \mathbb{R}$ by

$$f_p(u, v) = \begin{cases} c_f(p) & \text{if } (u, v) \text{ is on } p, \\ 0 & \text{otherwise} \end{cases}$$

Then, f_p is a flow in G_f with value $|f_p| = c_f(p) > 0$.

CLRS, 24.2-8

Exercise

Suppose that we redefine the residual network to disallow edges into s. Argue that the procedure Ford-Fulkerson still correctly computes a maximum flow.

CLRS, 24.2-9

Exercise

Suppose that both f and f' are flows in a flow network. Does the augmented flow $f \uparrow f'$ satisfy the flow conservation property? Does it satisfy the capacity constraint?

CLRS, 24.2-10

Exercise

Show how to find a maximum flow in a flow network by a sequence of at most $|E|$ augmenting paths. (Hint: Determine the paths after finding the maximum flow.)

CLRS, 24.2-11

Exercise

The edge connectivity of an undirected graph is the minimum number k of edges that must be removed to disconnect the graph. For example, the edge connectivity of a tree is 1, and the edge connectivity of a cyclic chain of vertices is 2. Show how to determine the edge connectivity of an undirected graph $G = (V, E)$ by running a maximum-flow algorithm on at most $|V|$ flow networks, each having $O(V + E)$ vertices and $O(E)$ edges.

CLRS, 24.2-12

Exercise

You are given a flow network G, where G contains edges entering the source s. Let f be a flow in G with $|f| \geq 0$ in which one of the edges (v, s) entering the source has $f(v, s) = 1$. Prove that there must exist another flow f' with $f'(v, s) = 0$ such that $|f| = |f'|$. Give an $O(E)$-time algorithm to compute f', given f and assuming that all edge capacities are integers.

CLRS, 24.2-13

Exercise

Suppose that you wish to find, among all minimum cuts in a flow network G with integer capacities, one that contains the smallest number of edges. Show how to modify the capacities of G to create a new flow network G' in which any minimum cut in G' is a minimum cut with the smallest number of edges in G.