Sidste gang

Aritmetriske rækker:

$$\sum_{i=1}^{n} (a+di) = (n+1)a + \frac{n(n+1)}{2} d$$

Eks: $\sum_{i=1}^{n} (1+2i) = (n+1)^2 = n^2+2n+1$

Dobbeltsummer:

$$\sum_{i=1}^{n} \sum_{j=1}^{i} j = \sum_{i=1}^{n} \frac{i(i+1)}{2} = \frac{1}{2}\sum_{i=1}^{n} i^2 + \frac{1}{2}\sum_{i=1}^{n} i$$

$$= \frac{1}{2} \frac{n(n+1)(2n+1)}{6} + \frac{1}{2} \frac{n(n+1)}{2}$$

$$= \frac{1}{2} \frac{n(n+1)(2n+4)}{6} = \frac{n(n+1)(n+2)}{6}$$

$$\sum_{i=1}^{10} \sum_{j=1}^{20} \sum_{k=1}^{5} (i+j+k) = \sum_{i=1}^{10} \sum_{j=1}^{20} \left(\sum_{k=1}^{5} (i+j) + \sum_{k=1}^{5} k \right)$$

$$= \sum_{i=1}^{10} \sum_{j=1}^{20} \left(5(i+j) + \frac{5 \cdot 6}{2} \right)$$

$$= 5 \cdot \sum_{i=1}^{10} \sum_{j=1}^{20} (i+j+3)$$

$$= 5 \sum_{i=1}^{10} \left(20i + \frac{20 \cdot 21}{2} + 20 \cdot 3 \right)$$

$$= 5 \cdot 20 \sum_{i=1}^{10} \left(i + \frac{21}{2} + 3 \right)$$

$$= 100 \left(\frac{10 \cdot 11}{2} + \frac{210}{2} + 30 \right)$$

$$= 100(55 + 105 + 30) = 19.000$$

Coupon Collector's Problem:

n forskellige klistermærker, 1 i hver pakke
I gennemsnit skal man købe nH_n pakker,
før man har alle n forskellige klistermærker.

$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n} \approx \ln n$

kaldes n'te harmoniske tal.

I dag: $\boxed{\text{Grafer}}$

Afsnit 10.1: Graftyper

$\boxed{\text{Simple grafer}}$

$$G = (V, E)$$

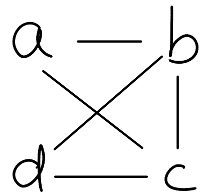
Graf — mængde af knuder — mængde af kanter

$e = \{u, v\} \in E$ forbinder u og v

e's endepunkter er u og v

u og v er naboer

Eks:



$V = \{a, b, c, d\}$

$E = \{\{a,b\}, \{a,c\}, \{b,c\}, \{b,d\}, \{c,d\}\}$

Eks. på veje/stier i G:

Længde (#kanter):		
3	a, b, c, d	Simple veje d.v.s. ingen knuder gentages
0	b	
1	b, c	
2	a, c, d	
5	b, c, a, b, c, d	

Eks. på kredse i G: (starter og slutter i samme knude)

a, b, c, a
b, c, d, b
} simple kredse

b, c, d, b, c, a, b

Simple orienterede grafer

Kanterne er ordnede par af knuder.

Eks:

$$\begin{tikzcd} a \arrow[r, bend left] \arrow[d] & b \arrow[l, bend left] \arrow[d] \\ d & c \arrow[l] \end{tikzcd}$$

$V = \{a, b, c, d\}$

$E = \{(a,b), (a,d), (b,a), (b,c), (c,d)\}$

Eks. på stier:

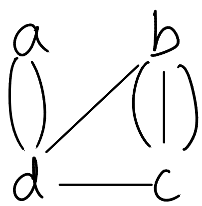
a, b, c
b, c } simple stier

a, b, a, d ikke simpel, da a gentages

Ikke sti: c, d, a

Multigrafer

Eks:

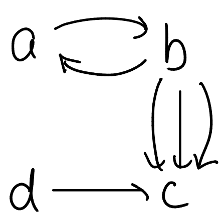


$V = \{a, b, c, d\}$

$E = \{ \{a,d\}, \{a,d\}, \{b,c\}, \{b,c\}, \{b,c\}, \{b,d\}, \{c,d\} \}$

multimængde

Eks: Orienteret multigraf

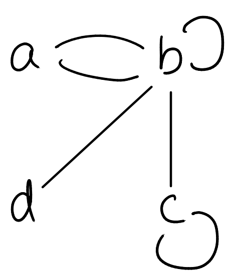


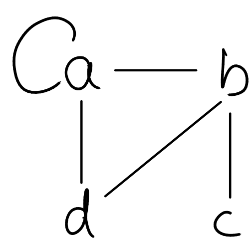
$V = \{a, b, c, d\}$

$E = \{ (a,b), (b,a), (b,c), (b,c), (b,c), (d,c) \}$

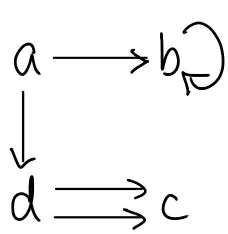
Pseudografer

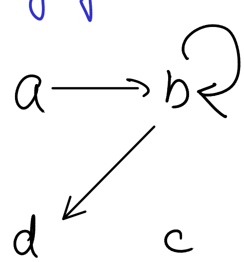
Eks:





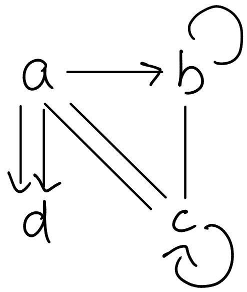
Eks: Orienteret pseudograf





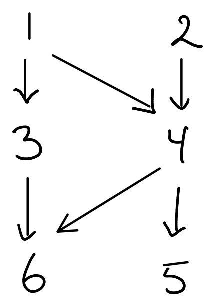
Blandede grafer

Eks:

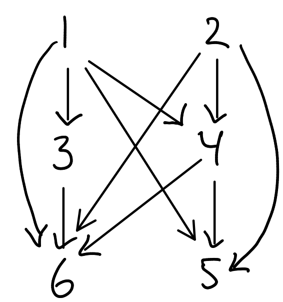


Eks 8: Præcedensgrafer

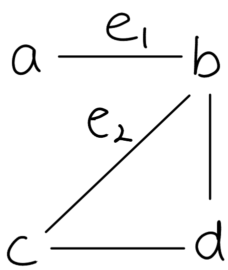
1. $a := 0$
2. $b := 1$
3. $c := a+1$
4. $d := b+a$
5. $e := d+1$
6. $e := c+d$



transitiv lukning



Afsnit 10.2:



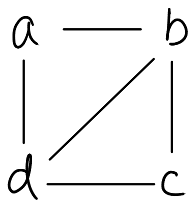
a og b er naboer / adjacente.
e_1 og e_2 er adjacente.
e_1 er incident til a og b.
e_1 forbinder a og b.

Graden af a er $\deg(a) = 1$, da a har 1 nabo.
$\deg(b) = 3$
$\deg(c) = \deg(d) = 2$

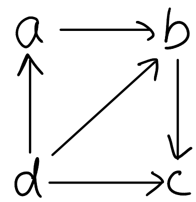
Afsnit 10.3

Repræsentation af grafer:

Adjacenslister

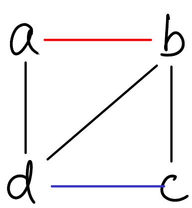


a: b, d
b: a, c, d
c: b, d
d: a, b, c

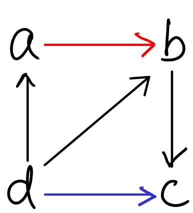


a: b
b: c
c:
d: a, b, c

Adjacensmatricer



$$
\begin{array}{c c}
 & \begin{array}{cccc} a & b & c & d \end{array} \\
\begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}
\end{array}
$$



$$
\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} = A
$$

$$A^2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \qquad A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\begin{tikzcd} a \arrow[r, shift left] \arrow[r, shift right] & b \arrow[d, shift left] \arrow[d, shift right] \\ d \arrow[u] \arrow[ur] \arrow[r] & c \end{tikzcd}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} = A^2$$

$A^2_{13} \geq 1$, hvis

$a_{11} = a_{13} = 1 \;\vee\; a_{12} = a_{23} = 1 \;\vee\; a_{13} = a_{33} = 1 \;\vee\; a_{14} = a_{34} = 1$

d.v.s. hvis

$(a,a), (a,c) \in E \;\vee\; (a,b), (b,c) \in E \;\vee\; (a,c), (c,c) \in E \;\vee$

$(a,d), (d,c) \in E$

$$A^3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\begin{tikzcd} a \arrow[r] & b \arrow[d] \\ d \arrow[u] \arrow[ur] \arrow[r] & c \end{tikzcd}$$

$$A^2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = A^3$$

$A^3_{43} = 1$, fordi $(d,a), (a,b), (b,c) \in E$

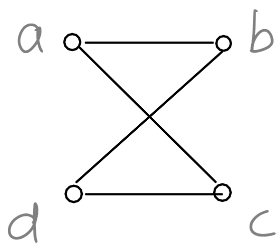
$a_{41} = a_{12} = a_{23} = 1 \implies$

$a^2_{42} = a_{23} = 1 \implies$

$a^3_{43} = 1$

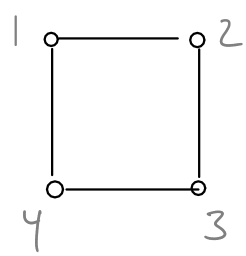
Isomorfi:

Eks



$\xrightarrow{f}$

a: b, c	1: 2, 4
b: a, d	2: 1, 3
c: a, d	3: 2, 4
d: b, c	4: 1, 3



1: 2, 4
2: 1, 3
3: 2, 4
4: 1, 3

Forskellige grafer ?

Def. 1

To simple grafer $G_1 = (V_1, E_1)$ og $G_2 = (V_2, E_2)$ er isomorfe, hvis $\exists$ bijektion $f : V_1 \to V_2$, så

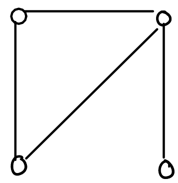
$$\{a, b\} \in E_1 \iff \{f(a), f(b)\} \in E_2,$$

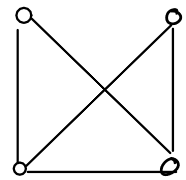
for alle $a, b \in V_1$.

De to grafer i eks. ovenfor er isomorfe:

$f(a) = 1$
$f(b) = 2$
$f(c) = 4$
$f(d) = 3$

Eks:





Isomorfe ?

Nej, der er ikke det samme #kanter.

Eks: