DM204, 2011
SCHEDULING, TIMETABLING AND ROUTING

Lecture 1

Introduction to Scheduling: Terminology and Classification

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Outline

1. Course Introduction

2. Scheduling
   - Definitions
   - Classification
   - Exercises
   - Schedules

3. Complexity Hierarchy

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Course Content

Scheduling (Manufacturing)

- Single and Parallel Machine Models
- Flow Shops and Flexible Flow Shops
- Job Shops
- Resource-Constrained Project Scheduling

Timetabling (Services)

- Interval Scheduling, Reservations
- Educational Timetabling
- Crew, Workforce and Employee Timetabling
- Transportation Timetabling

Vehicle Routing

- Capacited Vehicle Routing
- Vehicle Routing with Time Windows
- Rich Models

General Optimization Methods

- Mathematical Programming
- Constraint Programming
- Heuristics
- Problem Specific Algorithms (Dynamic Programming, Branch and Bound, ...)

Course Presentation

- Lecture plan and Schedule
- Communication tools
  - Course Public Web Site (WS) $\Leftrightarrow$ Blackboard (Bb)
    (public web site: http://www.imada.sdu.dk/~marco/DM204/)
  - Announcements (Bb)
  - Discussion board or Blog (Bb) not monitored
  - Personal email (Bb)
  - My office in working hours

Course Presentation

- Final Assessment (5 ECTS)
  - Oral exam: 30 minutes + 5 minutes defense project
    meant to assess the base knowledge
    $\rightsquigarrow$ based on a Case Portfolio
- Schedule:
  - Oral exam: in June, day to define

Course Material

- Literature
  - B1 Pinedo, M. Scheduling: Theory, Algorithms, and Systems Springer New York, 2008
    available online
  - B2 Pinedo, M. Planning and Scheduling in Manufacturing and Services Springer Verlag, 2005
    available online
  - B3 Toth, P. & Vigo, D. (ed.) The Vehicle Routing Problem SIAM Monographs on Discrete Mathematics and Applications, 2002
    photocopies
- Articles and photocopies available from the web site
- Lecture slides

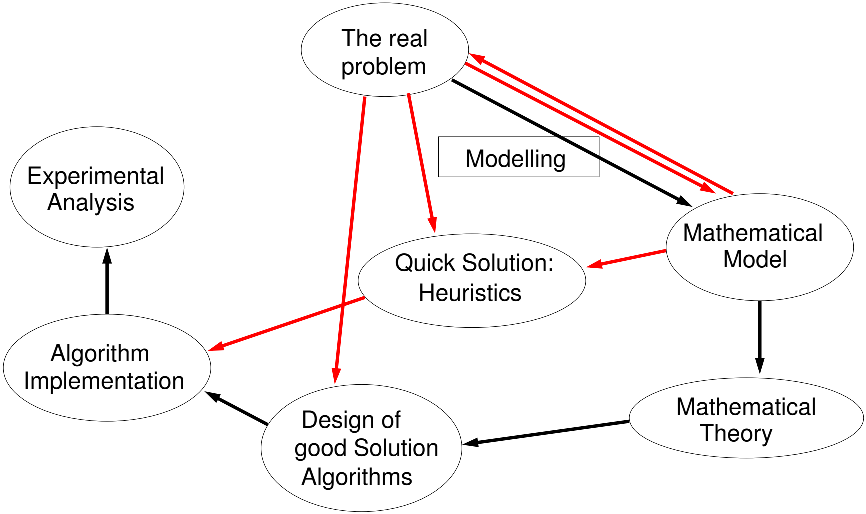
Course Goals

How to Tackle Real-life Optimization Problems:

- Formulate (mathematically) the problem
- Model the problem and recognize possible similar problems
- Search in the literature (or in the Internet) for:
  - complexity results (is the problem NP-hard?)
  - solution algorithms for original problem
  - solution algorithms for simplified problem
- Design solution algorithms and implement them
- Test experimentally with the goals of:
  - checking computational feasibility
  - configuring
  - comparing

Key ideas:
Decompose problems
Hybridize methods

The Problem Solving Cycle



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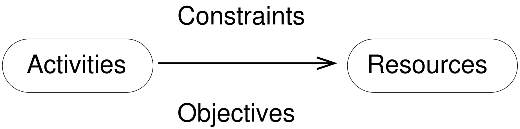
Scheduling

- Manufacturing
  - Project planning
  - Single, parallel machine and job shop systems
  - Flexible assembly systems
    Automated material handling (conveyor system)
  - Lot sizing
  - Supply chain planning
- Services
  - personnel/workforce scheduling
  - public transports

$\Rightarrow$ different models and algorithms

Problem Definition



Problem Definition

Given: a set of jobs $\mathcal{J} = \{J_1, \ldots, J_n\}$ to be processed by a set of machines $\mathcal{M} = \{M_1, \ldots, M_m\}$.

Task: Find a schedule, that is, a mapping of jobs to machines and processing times, that satisfies some constraints and is optimal w.r.t. some criteria.

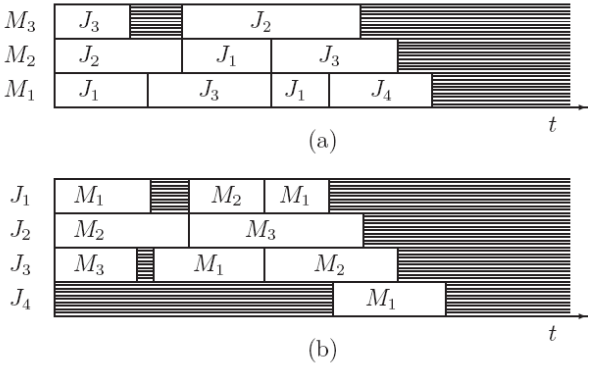
Notation:
n, j, k jobs
m, i, h machines

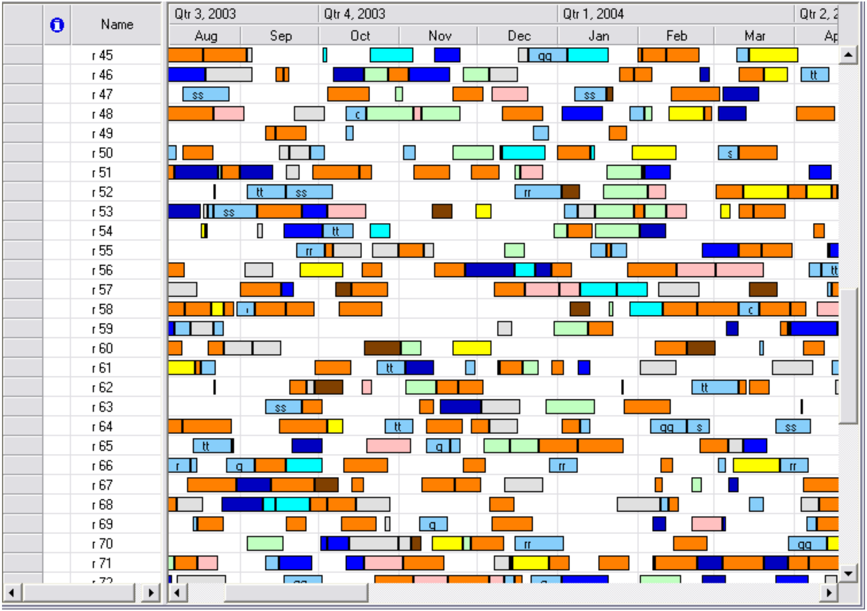
Visualization

Scheduling are represented by Gantt charts

(a) machine-oriented

(b) job-oriented

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Data Associated to Jobs

- Processing time p_{ij}
- Release date r_j
- Due date d_j (called deadline, if strict)
- Weight w_j
- Cost function $h_j(t)$ measures cost of completing J_j at t
- A job J_j may also consist of a number n_j of operations $O_{j1}, O_{j2}, \dots, O_{jn_j}$ and data for each operation.
- A set of machines $\mu_{jl} \subseteq \mathcal{M}$ associated to each operation
  - $|\mu_{jl}| = 1$ dedicated machines
  - $\mu_{jl} = \mathcal{M}$ parallel machines
  - $\mu_{jl} \subseteq \mathcal{M}$ multipurpose machines

Data that depend on the schedule

- Starting times S_{ij}
- Completion time C_{ij}, C_j

Problem Classification

A scheduling problem is described by a triplet $\alpha \mid \beta \mid \gamma$.

- α machine environment (one or two entries)
- β job characteristics (none or multiple entry)
- γ objective to be minimized (one entry)

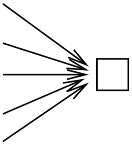
[R.L. Graham, E.L. Lawler, J.K. Lenstra, A.H.G. Rinnooy Kan (1979): Optimization and approximation in deterministic sequencing and scheduling: a survey, Ann. Discrete Math. 4, 287-326.]

$\alpha \mid \beta \mid \gamma$ Classification Scheme

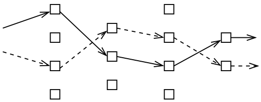
Machine Environment $\alpha_1\alpha_2 \mid \beta_1 \dots \beta_{13} \mid \gamma$

- single machine/multi-machine ($\alpha_1 = \alpha_2 = 1$ or $\alpha_2 = m$)
- parallel machines: identical ($\alpha_1 = P$), uniform p_j/v_i ($\alpha_1 = Q$), unrelated p_j/v_{ij} ($\alpha_1 = R$)
- multi operations models: Flow Shop ($\alpha_1 = F$), Open Shop ($\alpha_1 = O$), Job Shop ($\alpha_1 = J$), Mixed (or Group) Shop ($\alpha_1 = X$), Multi-processor task sched.

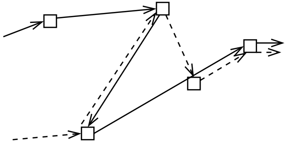
Single Machine



Flexible Flow Shop ($\alpha = FFc$)



Open, Job, Mixed Shop



$\alpha \mid \beta \mid \gamma$ Classification Scheme

Job Characteristics $\alpha_1\alpha_2 \mid \boldsymbol{\beta_1 \ldots \beta_{13}} \mid \gamma$

- $\beta_1 = \mathit{prmp}$ presence of preemption (resume or repeat)
- β_2 precedence constraints between jobs acyclic digraph $G = (V, A)$
  - $\beta_2 = \mathit{prec}$ if G is arbitrary
  - $\beta_2 = \{\mathit{chains}, \mathit{intree}, \mathit{outtree}, \mathit{tree}, \mathit{sp\text{-}graph}\}$
- $\beta_3 = r_j$ presence of release dates
- $\beta_4 = p_j = p$ preprocessing times are equal
- ($\beta_5 = d_j$ presence of deadlines)
- $\beta_6 = \{\mathit{s\text{-}batch}, \mathit{p\text{-}batch}\}$ batching problem
- $\beta_7 = \{s_{jk}, s_{jik}\}$ sequence dependent setup times

$\alpha \mid \beta \mid \gamma$ Classification Scheme

Job Characteristics (2) $\alpha_1\alpha_2 \mid \beta_1 \ldots \beta_{13} \mid \gamma$

- $\beta_8 = brkdwn$ machine breakdowns
- $\beta_9 = M_j$ machine eligibility restrictions (if $\alpha = Pm$)
- $\beta_{10} = prmu$ permutation flow shop
- $\beta_{11} = block$ presence of blocking in flow shop (limited buffer)
- $\beta_{12} = nwt$ no-wait in flow shop (limited buffer)
- $\beta_{13} = recrc$ recirculation in job shop

$\alpha \mid \beta \mid \gamma$ Classification Scheme

Objective (always $f(C_j)$) $\alpha_1\alpha_2 \mid \beta_1\beta_2\beta_3\beta_4 \mid \gamma$

- Lateness $L_j = C_j - d_j$
- Tardiness $T_j = \max\{C_j - d_j, 0\} = \max\{L_j, 0\}$
- Earliness $E_j = \max\{d_j - C_j, 0\}$
- Unit penalty $U_j = \begin{cases} 1 & \text{if } C_j > d_j \\ 0 & \text{otherwise} \end{cases}$

$\alpha \mid \beta \mid \gamma$ Classification Scheme

Objective $\alpha_1\alpha_2 \mid \beta_1\beta_2\beta_3\beta_4 \mid \gamma$

- Makespan: Maximum completion $C_{max} = \max\{C_1, \ldots, C_n\}$
  tends to max the use of machines
- Maximum lateness $L_{max} = \max\{L_1, \ldots, L_n\}$
- Total completion time $\sum C_j$ (flow time)
- Total weighted completion time $\sum w_j \cdot C_j$
  tends to min the av. num. of jobs in the system, ie, work in progress, or also the throughput time
- Discounted total weighted completion time $\sum w_j(1 - e^{-rC_j})$
- Total weighted tardiness $\sum w_j \cdot T_j$
- Weighted number of tardy jobs $\sum w_j U_j$

All regular functions (nondecreasing in $C_1, \ldots, C_n$) except E_i

$\alpha \mid \beta \mid \gamma$ Classification Scheme

Other Objectives

$\alpha_1\alpha_2 \mid \beta_1\beta_2\beta_3\beta_4 \mid \gamma$

Non regular objectives

- Min $\sum w_j' E_j + \sum w"_j T_j$ (just in time)
- Min waiting times
- Min set up times/costs
- Min transportation costs

Exercises

Gate Assignment at an Airport

- Airline terminal at a airport with dozes of gates and hundreds of arrivals each day.
- Gates and Airplanes have different characteristics
- Airplanes follow a certain schedule
- During the time the plane occupies a gate, it must go through a series of operations
- There is a scheduled departure time (due date)
- Performance measured in terms of on time departures.

Exercises

Scheduling Tasks in a Central Processing Unit (CPU)

- Multitasking operating system
- Schedule time that the CPU devotes to the different programs
- Exact processing time unknown but an expected value might be known
- Each program has a certain priority level
- Tasks are often sliced into little pieces. They are then rotated such that low priority tasks of short duration do not stay for ever in the system.
- Minimize expected time

Exercises

Paper bag factory

- Basic raw material for such an operation are rolls of paper.
- Production process consists of three stages: (i) printing of the logo, (ii) gluing of the side of the bag, (iii) sewing of one end or both ends.
- Each stage consists of a number of machines which are not necessarily identical.
- Each production order indicates a given quantity of a specific bag that has to be produced and shipped by a committed shipping date or due date.
- Processing times for the different operations are proportional to the number of bags ordered.
- There are setup times when switching over different types of bags (colors, sizes) that depend on the similarities between the two consecutive orders
- A late delivery implies a penalty that depends on the importance of the order or the client and the tardiness of the delivery.

Solutions

Distinction between

- sequence
- schedule
- scheduling policy

If no preemption allowed, schedule defined by vector $S = (S_i)$

Feasible schedule

A schedule is feasible if no two time intervals overlap on the same machine, and if it meets a number of problem specific constraints.

Optimal schedule

A schedule is optimal if it is feasible and it minimizes the given objective.

Classes of Schedules

Semi-active schedule

A feasible schedule is called semi-active if no operation can be completed earlier without changing the order of processing on any one of the machines. (local shift)

Active schedule

A feasible schedule is called active if it is not possible to construct another schedule by changing the order of processing on the machines and having at least one operation finishing earlier and no operation finishing later. (global shift without preemption)

Nondelay schedule

A feasible schedule is called nondelay if no machine is kept idle while an operation is waiting for processing. (global shift with preemption)

- There are optimal schedules that are nondelay for most models with regular objective function.
- There exists for $Jm||\gamma$ (γ regular) an optimal schedule that is active.
- nondelay $\Rightarrow$ active but active $\not\Rightarrow$ nondelay

Summary

- Scheduling Definitions (jobs, machines, Gantt charts)
- Classification
- Classes of schedules

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Complexity Hierarchy

Reduction

A search problem Π is (polynomially) reducible to a search problem Π' ($\Pi \longrightarrow \Pi'$) if there exists an algorithm $\mathcal{A}$ that solves Π by using a hypothetical subroutine $\mathcal{S}$ for Π' and except for $\mathcal{S}$ everything runs in polynomial time. [Garey and Johnson, 1979]

NP-hard

A search problem Π' is NP-hard if

1. it is in NP
2. there exists some NP-complete problem Π that reduces to Π'

In scheduling, complexity hierarchies describe relationships between different problems.

Ex: $1||\sum C_j \longrightarrow 1||\sum w_j C_j$

Interest in characterizing the borderline: polynomial vs NP-hard problems

Problems Involving Numbers

Partition

- **Input:** finite set A and a size $s(a) \in \mathbf{Z}^+$ for each $a \in A$
- **Question:** is there a subset $A' \subseteq A$ such that

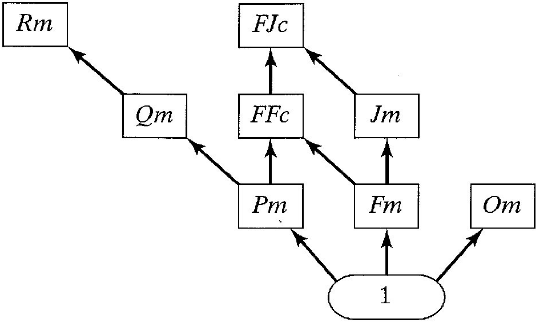
$$\sum_{a \in A'} s(a) = \sum_{a \in A - A'} s(a)?$$

3-Partition

- **Input:** set A of $3m$ elements, a bound $B \in \mathbf{Z}^+$, and a size $s(a) \in \mathbf{Z}^+$ for each $a \in A$ such that $B/4 < s(a) < B/2$ and such that $\sum_{a \in A} s(a) = mB$
- **Question:** can A be partitioned into m disjoint sets $A_1, \ldots, A_m$ such that for $1 \leq i \leq m$, $\sum_{a \in A_i} s(a) = B$ (note that each A_i must therefore contain exactly three elements from A)?

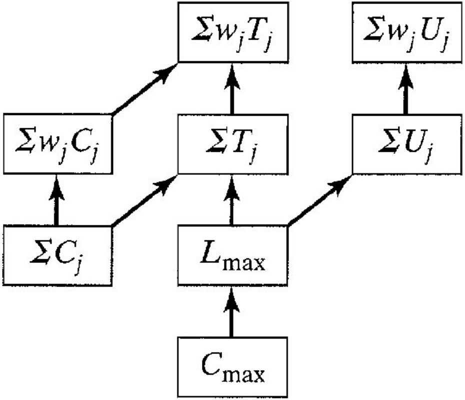
Complexity Hierarchy

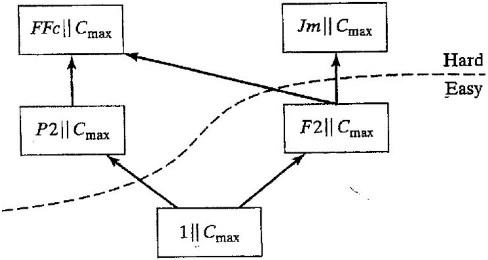
Elementary reductions for machine environment

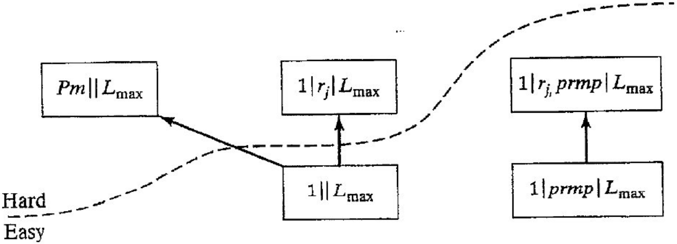


Complexity Hierarchy

Elementary reductions for regular objective functions




$FFc||C_{max}$
$Jm||C_{max}$
Hard
Easy
$P2||C_{max}$
$F2||C_{max}$
$1||C_{max}$


$Pm||L_{max}$
$1|r_j|L_{max}$
$1|r_j, prmp|L_{max}$
Hard
Easy
$1||L_{max}$
$1|prmp|L_{max}$

Polynomial time solvable problems

SINGLE MACHINE	PARALLEL MACHINES	SHOPS
$1 \mid r_j, p_j = 1, prec \mid \sum C_j$	$P2 \mid p_j = 1, prec \mid L_{\max}$	$O2 \mid\mid C_{\max}$
$1 \mid r_j, prmp \mid \sum C_j$	$P2 \mid p_j = 1, prec \mid \sum C_j$	
$1 \mid tree \mid \sum w_j C_j$		$Om \mid r_j, prmp \mid L_{\max}$
	$Pm \mid p_j = 1, tree \mid C_{\max}$	
$1 \mid prec \mid L_{\max}$	$Pm \mid prmp, tree \mid C_{\max}$	$F2 \mid block \mid C_{\max}$
$1 \mid r_j, prmp, prec \mid L_{\max}$	$Pm \mid p_j = 1, outtree \mid \sum C_j$	$F2 \mid nwt \mid C_{\max}$
	$Pm \mid p_j = 1, intree \mid L_{\max}$	
$1 \mid\mid \sum U_j$	$Pm \mid prmp, intree \mid L_{\max}$	$Fm \mid p_{ij} = p_j \mid \sum C_j$
$1 \mid r_j, prmp \mid \sum U_j$		$Fm \mid p_{ij} = p_j \mid L_{\max}$
$1 \mid r_j, p_j = 1 \mid \sum w_j U_j$	$Q2 \mid prmp, prec \mid C_{\max}$	$Fm \mid p_{ij} = p_j \mid \sum U_j$
	$Q2 \mid r_j, prmp, prec \mid L_{\max}$	
$1 \mid r_j, p_j = 1 \mid \sum w_j T_j$		$J2 \mid\mid C_{\max}$
	$Qm \mid r_j, p_j = 1 \mid C_{\max}$	
	$Qm \mid p_j = 1, M_j \mid C_{\max}$	
	$Qm \mid r_j, p_j = 1 \mid \sum C_j$	
	$Qm \mid prmp \mid \sum C_j$	
	$Qm \mid p_j = 1 \mid \sum w_j C_j$	
	$Qm \mid p_j = 1 \mid L_{\max}$	
	$Qm \mid prmp \mid \sum U_j$	
	$Qm \mid p_j = 1 \mid \sum w_j U_j$	
	$Qm \mid p_j = 1 \mid \sum w_j T_j$	
	$Rm \mid\mid \sum C_j$	
	$Rm \mid r_j, prmp \mid L_{\max}$	

NP-hard problems in the ordinary sense

SINGLE MACHINE	PARALLEL MACHINES	SHOPS
$1 \mid\mid \sum w_j U_j$ (*) $1 \mid r_j, prmp \mid \sum w_j U_j$ (*) $1 \mid\mid \sum T_j$ (*)	$P2 \mid\mid C_{\max}$ (*) $P2 \mid r_j, prmp \mid \sum C_j$ $P2 \mid\mid \sum w_j C_j$ (*) $P2 \mid r_j, prmp \mid \sum U_j$ $Pm \mid prmp \mid \sum w_j C_j$ $Qm \mid\mid \sum w_j C_j$ (*) $Rm \mid r_j \mid C_{\max}$ (*) $Rm \mid\mid \sum w_j U_j$ (*) $Rm \mid prmp \mid \sum w_j U_j$	$O2 \mid prmp \mid \sum C_j$ $O3 \mid\mid C_{\max}$ $O3 \mid prmp \mid \sum w_j U_j$

Strongly NP-hard problems

SINGLE MACHINE	PARALLEL MACHINES	SHOPS
$1 \mid s_{jk} \mid C_{\max}$ $1 \mid r_j \mid \sum C_j$ $1 \mid prec \mid \sum C_j$ $1 \mid r_j, prmp, tree \mid \sum C_j$ $1 \mid r_j, prmp \mid \sum w_j C_j$ $1 \mid r_j, p_j = 1, tree \mid \sum w_j C_j$ $1 \mid p_j = 1, prec \mid \sum w_j C_j$ $1 \mid r_j \mid L_{\max}$ $1 \mid r_j \mid \sum U_j$ $1 \mid p_j = 1, chains \mid \sum U_j$ $1 \mid r_j \mid \sum T_j$ $1 \mid p_j = 1, chains \mid \sum T_j$ $1 \mid\mid \sum w_j T_j$	$P2 \mid chains \mid C_{\max}$ $P2 \mid chains \mid \sum C_j$ $P2 \mid prmp, chains \mid \sum C_j$ $P2 \mid p_j = 1, tree \mid \sum w_j C_j$ $R2 \mid prmp, chains \mid C_{\max}$	$F2 \mid r_j \mid C_{\max}$ $F2 \mid r_j, prmp \mid C_{\max}$ $F2 \mid\mid \sum C_j$ $F2 \mid prmp \mid \sum C_j$ $F2 \mid\mid L_{\max}$ $F2 \mid prmp \mid L_{\max}$ $F3 \mid\mid C_{\max}$ $F3 \mid prmp \mid C_{\max}$ $F3 \mid nwt \mid C_{\max}$ $O2 \mid r_j \mid C_{\max}$ $O2 \mid\mid \sum C_j$ $O2 \mid prmp \mid \sum w_j C_j$ $O2 \mid\mid L_{\max}$ $O3 \mid prmp \mid \sum C_j$ $J2 \mid rcrc \mid C_{\max}$ $J3 \mid p_{ij} = 1, rcrc \mid C_{\max}$

Web Archive

Complexity results for scheduling problems
by Peter Brucker and Sigrid Knust
http://www.mathematik.uni-osnabrueck.de/research/OR/class/