DM204 – Spring 2011

Scheduling, Timetabling and Routing

Lecture 7

Timetabling: Reservations and Education

Marco Chiarandini

Department of Mathematics & Computer Science
University of Southern Denmark

Outline

Course Overview

- ✔ Scheduling
  - ✔ Classification
  - ✔ Complexity issues
  - ✔ Single Machine
  - ✔ Parallel Machine
  - ✔ Flow Shop and Job Shop
  - ✔ Resource Constrained Project Scheduling Model

- Timetabling
  - ✔ Sport Timetabling
  - Reservations and Education
  - University Timetabling
  - Crew Scheduling
  - Public Transports

- Vechicle Routing
  - Capacited Models
  - Time Windows models
  - Rich Models

Timetabling

- Educational Timetabling
  - School/Class timetabling
  - University/Course timetabling
  - curriculum planning
  - project assignment
- Personnel/Employee timetabling
  - Crew scheduling
  - Crew rostering
- Transport Timetabling
- Sports Timetabling
- Communication Timetabling

Outline

1. Reservations without slack
2. Reservations with slack
3. Timetabling with one Operator
4. Timetabling with Operators
5. Educational Timetabling

Reservations without slack
Interval Scheduling

Given:

- m parallel machines (resources)
- n activities
- r_j starting times (integers),
  d_j termination (integers),
  w_j or w_{ij} weight,
  M_j eligibility
- without slack $p_j = d_j - r_j$

Task: Maximize weight of assigned activities

Examples: Hotel room reservation, Car rental

Polynomially solvable cases

1. $p_j = 1$

Solve an assignment problem at each time slot

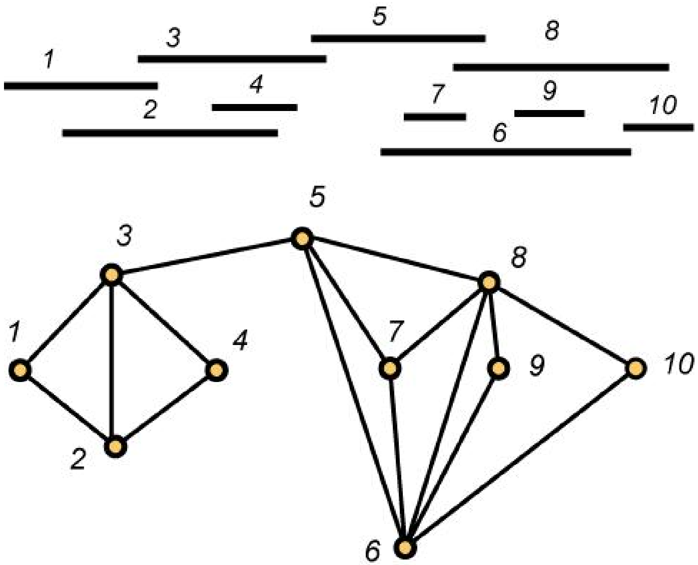
2. $w_j = 1$, $M_j = M$, Obj. minimize resources used

- Corresponds to coloring interval graphs with minimal number of colors
- Optimal greedy algorithm (First Fit):

order $r_1 \leq r_2 \leq \ldots \leq r_n$

Step 1 assign resource 1 to activity 1

Step 2 **for** j from 2 to n **do**
Assume k resources have been used.
Assign activity j to the resource with minimum feasible value from $\{1, \ldots, k+1\}$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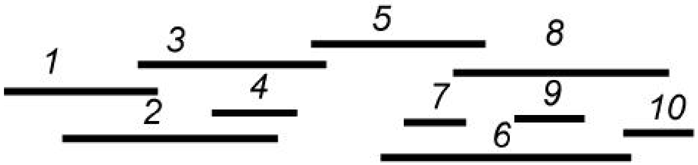
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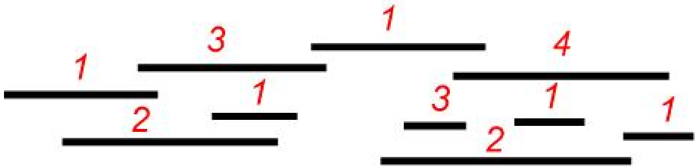
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3. $w_j = 1$, $M_j = M$, Obj. maximize activities assigned

- Corresponds to coloring max # of vertices in interval graphs with k colors
- Optimal k-coloring of interval graphs:

order $r_1 \leq r_2 \leq \ldots \leq r_n$

$J = \emptyset$, $j = 1$

Step 1 if a resource is available at time r_j then assign activity j to that resource;
include j in J; go to Step 3

Step 2 Else, select j^* such that $C_{j^*} = \max_{j \in J} C_j$
if $C_j = r_j + p_j > C_{j^*}$ go to Step 3
else remove j^* from J, assign j in J

Step 3 **if** $j = n$ STOP **else** $j = j + 1$ go to Step 1

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Reservations with Slack

Given:

- m parallel machines (resources)
- n activities
- r_j starting times (integers),
  d_j termination (integers),
  w_j or w_{ij} weight,
  M_j eligibility
- with slack $p_j \leq d_j - r_j$

Task: Maximize weight of assigned activities

Heuristics

Most constrained variable, least constraining value heuristic

$|M_j|$ indicates how much constrained an activity is

ν_{it}: # activities that can be assigned to i in $[t-1, t]$

Select activity j with smallest $I_j = f\left(\frac{w_j}{p_j}, |M_j|\right)$

Select resource i with smallest $g(\nu_{i,t+1}, \ldots, \nu_{i,t+p_j})$ (or discard j if no place free for j)

Examples for f and g:

$$f\left(\frac{w_j}{p_j}, |M_j|\right) = \frac{|M_j|}{w_j/p_j}$$

$$g(\nu_{i,t+1}, \ldots, \nu_{i,t+p_j}) = \max(\nu_{i,t+1}, \ldots, \nu_{i,t+p_j})$$

$$g(\nu_{i,t+1}, \ldots, \nu_{i,t+p_j}) = \sum_{l=1}^{p_j} \frac{\nu_{i,t+l}}{p_j}$$

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Timetabling with one Operator

There is only one type of operator that processes all the activities

Example:

- A contractor has to complete n activities.
- The duration of activity j is p_j
- Each activity requires a crew of size W_j.
- The activities are not subject to precedence constraints.
- The contractor has W workers at his disposal
- His objective is to complete all n activities in minimum time.

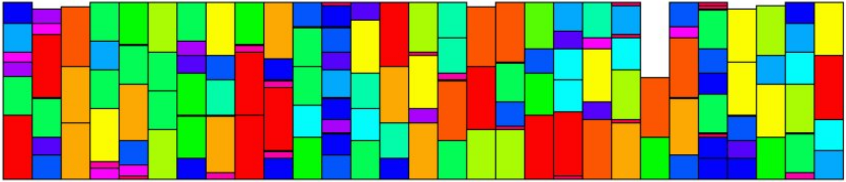
- RCPSP Model
- If p_j all the same ➜ Bin Packing Problem (still NP-hard)

Example: Exam scheduling

- Exams in a college with same duration.
- The exams have to be held in a gym with W seats.
- The enrollment in course j is W_j and
- all W_j students have to take the exam at the same time.
- The goal is to develop a timetable that schedules all n exams in minimum time.
- Each student has to attend a single exam.

- Bin Packing model
- In the more general (and realistic) case it is a RCPSP

Heuristics for Bin Packing



- Construction Heuristics
  - Best Fit Decreasing (BFD)
  - First Fit Decreasing (FFD) $C_{max}(FFD) \leq \frac{11}{9} C_{max}(OPT) + \frac{6}{9}$

- Local Search: [Alvim and Aloise and Glover and Ribeiro, 1999]

  Step 1: remove one bin and redistribute items by BFD

  Step 2: if infeasible, re-make feasible by redistributing items for pairs of bins, such that their total weights becomes equal (number partitioning problem)

[Levine and Ducatelle, 2004]

The solution before local search (the bin capacity is 10):

The bins: | 3 3 3 | 6 2 1 | 5 2 | 4 3 | 7 2 | 5 4 |

Open the two smallest bins:

Remaining: | 3 3 3 | 6 2 1 | 7 2 | 5 4 |

Free items: 5, 4, 3, 2

Try to replace 2 current items by 2 free items, 2 current by 1 free or 1 current by 1 free:

First bin: $3\ 3\ 3 \rightarrow 3\ 5\ 2$ new free: 4, 3, 3, 3

Second bin: $6\ 2\ 1 \rightarrow 6\ 4$ new free: 3, 3, 3, 2, 1

Third bin: $7\ 2 \rightarrow 7\ 3$ new free: 3, 3, 2, 2, 1

Fourth bin: 5 4 stays the same

Reinsert the free items using FFD:

Fourth bin: $5\ 4 \rightarrow 5\ 4\ 1$

Make new bin: 3 3 2 2

Final solution: | 3 5 2 | 6 4 | 7 3 | 5 4 1 | 3 3 2 2 |

Outline

Timetabling with Operators

- There are several operators and activities can be done by an operator only if he is available
- Two activities that share an operator cannot be scheduled at the same time

Examples:

- aircraft repairs
- scheduling of meetings (people ➜ operators; resources ➜ rooms)
- exam scheduling (students may attend more than one exam ➜ operators)

If $p_j = 1$ ➜ Graph-Vertex Coloring (still NP-hard)

Mapping to Graph-Vertex Coloring

- activities ➜ vertices
- if 2 activities require the same operators ➜ edges
- time slots ➜ colors
- feasibility problem (if # time slots is fixed)
- optimization problem

DSATUR heuristic for Graph-Vertex Coloring

saturation degree: number of differently colored adjacent vertices

set of empty color classes $\{C_1, \ldots, C_k\}$, where $k = |V|$

Sort vertices in decreasing order of their degrees

Step 1 A vertex of maximal degree is inserted into C_1.

Step 2 The vertex with the maximal saturation degree is chosen and inserted according to the greedy heuristic (first feasible color). Ties are broken preferring vertices with the maximal number of adjacent, still uncolored vertices; if further ties remain, they are broken randomly.

Outline

Educational timetabling process

Phase:	Planning	Scheduling	Dispatching
Horizon:	Long Term	Timetable Period	Day of Operation
Objective:	Service Level	Feasibility	Get it Done
Steps:	Manpower, Curriculum, Equipment	Quarterly Timetabling, Project assignment, student sectioning	Repair