

# Outline

DM811  
Heuristics for Combinatorial Optimization

## Lecture 11 Examples

Marco Chiarandini

Department of Mathematics & Computer Science  
University of Southern Denmark

1. Recap.
2. Examples
  - TSP
  - Graph Coloring
  - Indirect Solution Representation

2

# Outline

Recap.  
Examples

## Summary: Local Search Algorithms (as in [Hoos, Stützle, 2005])

Recap.  
Examples

1. Recap.

2. Examples

TSP  
Graph Coloring  
Indirect Solution Representation

For given problem instance  $\pi$ :

1. search space  $S_\pi$
2. neighborhood relation  $\mathcal{N}_\pi \subseteq S_\pi \times S_\pi$
3. evaluation function  $f_\pi : S \rightarrow \mathbf{R}$
4. set of memory states  $M_\pi$
5. initialization function  $\text{init} : \emptyset \rightarrow S_\pi \times M_\pi$
6. step function  $\text{step} : S_\pi \times M_\pi \rightarrow S_\pi \times M_\pi$
7. termination predicate  $\text{terminate} : S_\pi \times M_\pi \rightarrow \{\top, \perp\}$

# Outline

After implementation and test of the above components, improvements in efficiency (ie, computation time) can be achieved by:

- A. fast delta evaluation
- B. neighborhood pruning
- C. clever use of data structures

Improvements in quality can be achieved by:

- D. application of a metaheuristic
- E. definition of a larger neighborhood

1. Recap.

2. Examples

- TSP
- Graph Coloring
- Indirect Solution Representation

## Asymmetric TSP into Symmetric TSP

## Local Search for Graph coloring

How to encode an asymmetric TSP into a symmetric TSP?

Different choices for the [candidate solutions](#), [neighborhood structures](#) and [evaluation function](#) define different approaches to the problem

<i>k</i> -fixed	complete	proper	
<i>k</i> -fixed	partial	proper	+++
<i>k</i> -fixed	complete	unproper	+++
<i>k</i> -fixed	partial	unproper	-
<i>k</i> -variable	complete	proper	++
<i>k</i> -variable	partial	proper	-
<i>k</i> -variable	complete	unproper	++
<i>k</i> -variable	partial	unproper	-

Polynomial time simplifications

$k$ -coloring ( $k$  fixed)

- Remove under-constrained nodes
- Remove subsumed nodes
- Merge nodes that must have the same color

11

Total Weighted Completion Time on Unrelated Parallel Machines Problem

**Input:** A set of jobs  $J$  to be processed on a set of parallel machines  $M$ . Each job  $j \in J$  has a weight  $w_j$  and processing time  $p_{ij}$  that depends on the machine  $i \in M$  on which it is processed.

**Task:** Find a schedule of the jobs on the machines such that the sum of weighted completion time of the jobs is minimal.

14

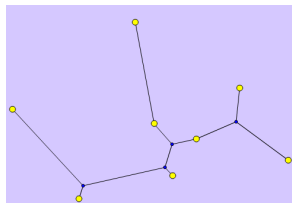
## Steiner Tree

Steiner Tree Problem

**Input:** A graph  $G = (V, E)$ , a weight function  $\omega : E \mapsto \mathbf{N}$ , and a subset  $U \subseteq V$ .

**Task:** Find a Steiner tree, that is, a subtree  $T = (V_T, E_T)$  of  $G$  that includes all the vertices of  $U$  and such that the sum of the weights of the edges in the subtree is minimal.

Vertices in  $U$  are the special vertices and vertices in  $S = V \setminus U$  are Steiner vertices.



15