DM811 – Autumn 2011

Heuristics for Combinatorial Optimization

Compendium

Basic Concepts in Algorithmics

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Outline

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1. Basic Concepts from Previous Courses
   - Graphs
   - Notation and runtime
   - Machine model
   - Pseudo-code
   - Computational Complexity
   - Analysis of Algorithms

Graphs

Graphs are combinatorial structures useful to model several applications

Terminology:

- $G = (V, E)$, $E \subseteq V \times V$, vertices, edges, $n = |V|$, $m = |E|$, digraphs, undirected graphs, subgraph, induced subgraph

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- parent, children, sibling, height, depth

Representing Graphs

Operations:

- Access associated information (NodeArray, EdgeArray, Hashes)
- Navigation: access outgoing edges
- Edge queries: given u and v is there an edge?
- Update: add remove edges, vertices

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How to choose?

- it depends on the graphs and the application
- if time and space not crucial no need to customize the structures
- use interfaces that make easy to change the data structure
- libraries offer different choices (Boost, Java `jdsl.graph`)

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Motivations

Questions:

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2. How hard, computationally, is a given a problem to solve using the most efficient algorithm for that problem?

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1. Asymptotic notation, running time bounds
   Approximation theory

2. Complexity theory

Asymptotic notation

$n \in \mathbf{N}$ instance size

max time	worst case	$T(n) = \max\{T(\pi) \;:\; \pi \in \Pi_n\}$
average time	average case	$T(n) = \frac{1}{\lvert\Pi_n\rvert}\{\sum_\pi T(\pi) \;:\; \pi \in \Pi_n\}$
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Growth rate or asymptotic analysis

$f(n)$ and $g(n)$ same growth rate if $c \le \frac{f(n)}{g(n)} \le d$ for n large

$f(n)$ grows faster than $g(n)$ if $f(n) \ge c \cdot g(n)$ for all c and n large

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big O	$O(f) = \{g(n) \ : \ \exists c > 0, \forall n > n_0 \ : \ g(n) \leq c \cdot f(n)\}$
big omega	$\Omega(f) = \{g(n) \ : \ \exists c > 0, \forall n > n_0 \ : \ g(n) \geq c \cdot f(n)\}$
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(little o	$o(f) = \{g \ : \ g$ grows strictly more slowly$\}$)

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For asymptotic analysis we use RAM machine

- sequential, single processor unit
- all memory access take same amount of time

It is an abstraction from machine architecture: it ignores caches, memories hierarchies, parallel processing (SIMD, multi-threading), etc.

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Total execution of a program = total number of instructions executed

We are not interested in constant and lower order terms

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We express algorithms in natural language and mathematical notation, and in pseudo-code, which is an abstraction from programming languages C, C++, Java, etc.

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Programs must be correct.
Certifying algorithm: computes a certificate for a post condition (without increasing asymptotic running time)

Good Algorithms

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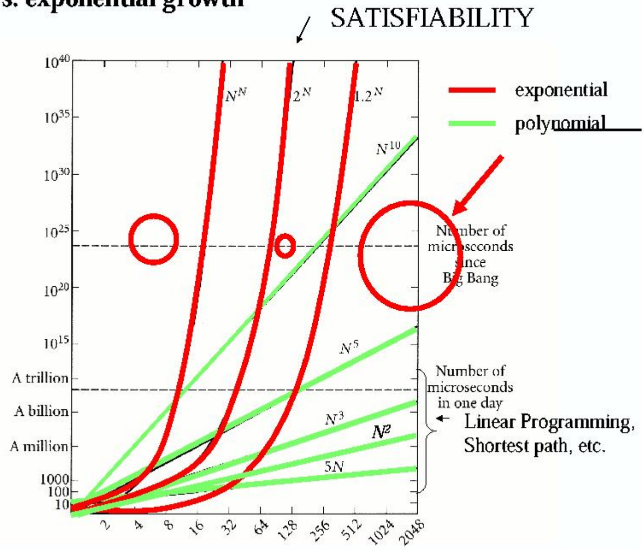
There are problems for which no polytime algorithm is known.
This course is about those problems.

Complexity theory classifies problems

Polynomial vs. exponential growth

(Harel 2000)



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- a search problem Π' is (polynomially) reducible to Π ($\Pi' \longrightarrow \Pi$) if there exists an algorithm $\mathcal{A}$ that solves Π' by using a hypothetical subroutine $\mathcal{S}$ for Π and except for $\mathcal{S}$ everything runs in polynomial time.

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- If Π satisfies property 2, but not necessarily property 1, we say that it is NP-hard:

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- *NP*-hard: At least as difficult as the most difficult problems in *NP*, but possibly not in *NP* (*i.e.*, may have even worse complexity than *NP*-complete problems).

SAT Problem
Satisfiability problem in propositional logic

Definitions:

- Formula in propositional logic: well-formed string that may contain
  - propositional variables $x_1, x_2, \ldots, x_n$;
  - truth values $\top$ ('true'), $\bot$ ('false');
  - operators $\neg$ ('not'), $\wedge$ ('and'), $\vee$ ('or');
  - parentheses (for operator nesting).

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- Model (or satisfying assignment) of a formula F: Assignment of truth values to the variables in F under which F becomes true (under the usual interpretation of the logical operators)

- Formula F is satisfiable iff there exists at least one model of F, unsatisfiable otherwise.

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- **Given:** Formula F in propositional logic
- **Task:** Find an assignment of truth values to variables in F that renders F true, or decide that no such assignment exists.

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SAT: A simple example

- **Given:** Formula $F := (x_1 \vee x_2) \wedge (\neg x_1 \vee \neg x_2)$
- **Task:** Find an assignment of truth values to variables x_1, x_2 that renders F true, or decide that no such assignment exists.

Definitions:

- A formula is in conjunctive normal form (CNF) iff it is of the form

  $$\bigwedge_{i=1}^{m} \bigvee_{j=1}^{k_i} l_{ij} = (l_{11} \vee \ldots \vee l_{1k_1}) \wedge \ldots \wedge (l_{m1} \vee \ldots \vee l_{mk_m})$$

  where each literal l_{ij} is a propositional variable or its negation. The disjunctions $c_i = (l_{i1} \vee \ldots \vee l_{ik_i})$ are called clauses.

- A formula is in k-CNF iff it is in CNF and all clauses contain exactly k literals (*i.e.*, for all i, $k_i = k$).

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- In many cases, the restriction of SAT to CNF formulae is considered.
- For every propositional formula, there is an equivalent formula in 3-CNF.

Example:

$$
\begin{aligned}
F := \quad & \wedge (\neg x_2 \vee x_1) \\
& \wedge (\neg x_1 \vee \neg x_2 \vee \neg x_3) \\
& \wedge (x_1 \vee x_2) \\
& \wedge (\neg x_4 \vee x_3) \\
& \wedge (\neg x_5 \vee x_3)
\end{aligned}
$$

- F is in CNF.
- Is F satisfiable?

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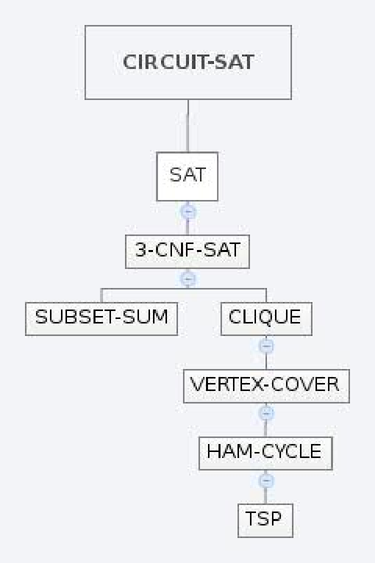
- F is in CNF.
- Is F satisfiable?
  Yes, *e.g.*, $x_1 := x_2 := \top$, $x_3 := x_4 := x_5 := \bot$ is a model of F.

MAX-SAT (optimization problem)

Which is the maximal number of clauses satisfiable in a propositional logic formula F?

NP-Completeness Proofs



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- TSP on Euclidean instances is *NP*-hard
  but not where all vertices lie on a circle.

An online compendium on the computational complexity of optimization problems:
http://www.nada.kth.se/~viggo/problemlist/compendium.html

Outline

Theoretical Analysis

- Worst-case analysis (runtime and quality):
  worst performance of algorithms over all possible instances
- Probabilistic analysis (runtime):
  average-case performance over a given probability distribution of instances
- Average-case (runtime):
  overall possible instances for randomized algorithms
- Asymptotic convergence results (quality)
- Approximation of optimal solutions:
  sometimes possible in polynomial time (*e.g.*, Euclidean TSP),
  but in many cases also intractable (*e.g.*, general TSP);
- Domination
- Algorithm invariance

Approximation Algorithms

Definition: Approximation Algorithms

An algorithm $\mathcal{A}$ is said to be a δ-approximation algorithm if it runs in polynomial time and for every problem instance π with optimal solution value $OPT(\pi)$

$$\text{minimization:} \quad \frac{\mathcal{A}(\pi)}{OPT(\pi)} \leq \delta \quad \delta \geq 1$$

$$\text{maximization:} \quad \frac{\mathcal{A}(\pi)}{OPT(\pi)} \geq \delta \quad \delta \leq 1$$

(δ is called *worst case bound*, *worst case performance*, *approximation factor*, *approximation ratio*, *performance bound*, *performance ratio*, *error ratio*)

Approximation Algorithms

Definition: Polynomial approximation scheme

A family of approximation algorithms for a problem Π, $\{\mathcal{A}_\epsilon\}_\epsilon$, is called a polynomial approximation scheme (PAS), if algorithm $\mathcal{A}_\epsilon$ is a $(1+\epsilon)$-approximation algorithm and its running time is polynomial in the size of the input for each fixed ϵ

Definition: Fully polynomial approximation scheme

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Useful Graph Algorithms

- Breadth first, depth first search, traversal
- Transitive closure
- Topological sorting
- (Strongly) connected components
- Shortest Path
- Minimum Spanning Tree
- Matching

Randomized Algorithms

Most often algorithms are randomized. Why?

Randomized Algorithms

Most often algorithms are randomized. Why?

- possibility of gains from re-runs
- adversary argument
- structural simplicity for comparable average performance,
- speed up,
- avoiding loops in the search
- ...

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Their running time depends on the random choices made.
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Las Vegas algorithm: it always gives the correct result but in random runtime (with finite expected value).

Monte Carlo algorithm: the result is not guaranteed correct. Typically halted due to bouned resources.

Randomized Heuristics

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both solution quality and runtime are random variables.

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We distinguish:

- single-pass heuristics (denoted $\mathcal{A}^{\dashv}$): have an embedded termination, for example, upon reaching a certain state
(generalized optimization Las Vegas algorithms [B2])

- asymptotic heuristics (denoted $\mathcal{A}^{\infty}$): do not have an embedded termination and they might improve their solution asymptotically
(both probabilistically approximately complete
and essentially incomplete [B2])