DM826 – Spring 2012

Modeling and Solving Constrained Optimization Problems

Exercises

Set Variables
SONET Problem

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[*Partly based on slides by Stefano Gualandi, Politecnico di Milano*]

Sonet **problem**

Optical fiber network design

Sonet problem

Input: weighted undirected demand graph $G = (N, E; d)$, where each node $u \in N$ represents a client and weighted edges $(u, v) \in E$ correspond to traffic demands of a pair of clients.

Two nodes can communicate, only if they join the same ring; nodes may join more than one ring. We must respect:

- maximum number of rings r
- maximum number of clients per ring a
- maximum bandwidth capacity of each ring c

Task: find a topology that minimizes the sum, over all rings, of the number of nodes that join each ring while clients' traffic demands are met.

Sonet **problem**

Sonet problem

A solution of the SONET problem is an assignment of rings to nodes and of capacity to demands such that

- all demands of each client pairs are satisfied;
- the ring traffic does not exceed the bandwidth capacity;
- at most r rings are used;
- at most a ADMs on each ring;
- the total number of ADMs used is minimized.

Sonet: **variables**

- Set variable X_i represents the set of nodes assigned to ring i
- Set variable Y_u represents the set of rings assigned to node u
- Integer variable Z_{ie} represents the amount of bandwidth assigned to demand pair e on ring i.

Sonet: **model**

$$
\begin{aligned}
\min \quad & \sum_{i \in R} |X_i| & \\
\text{s.t.} \quad & |Y_u \cap Y_v| \geq 1, & \forall (u,v) \in E, \\
& Z_{i,(u,v)} > 0 \Rightarrow i \in (Y_u \cap Y_v), & \forall i \in R, (u,v) \in E, \\
& Z_{ie} = d(e), & \forall e \in E, \\
& u \in X_i \Leftrightarrow i \in Y_u, & \forall \in R, u \in N, \\
& |X_i| \leq a, & \forall i \in R \\
& \sum_{e \in E} Z_{ie} \leq c, & \forall i \in R. \\
& X_i \preceq X_j, & \forall i, j \in R : i < j.
\end{aligned}
$$