DM204, 2011
SCHEDULING, TIMETABLING AND ROUTING

Lecture 1
Introduction to Scheduling: Terminology and Classification

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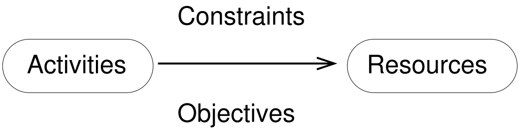
Problem Definition



Problem Definition

Given: a set of jobs $\mathcal{J} = \{J_1, \ldots, J_n\}$ to be processed by a set of machines $\mathcal{M} = \{M_1, \ldots, M_m\}$.

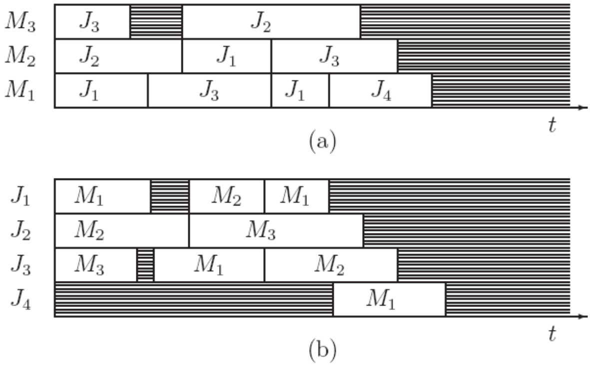
Task: Find a schedule, that is, a mapping of jobs to machines and processing times, that satisfies some constraints and is optimal w.r.t. some criteria.

Notation:
n, j, k jobs
m, i, h machines

Visualization

Scheduling are represented by Gantt charts

(a) machine-oriented
(b) job-oriented



Data Associated to Jobs

- Processing time p_{ij}
- Release date r_j
- Due date d_j (called deadline, if strict)
- Weight w_j
- Cost function $h_j(t)$ measures cost of completing J_j at t
- A job J_j may also consist of a number n_j of operations $O_{j1}, O_{j2}, \ldots, O_{jn_j}$ and data for each operation.
- A set of machines $\mu_{jl} \subseteq \mathcal{M}$ associated to each operation
  - $|\mu_{jl}| = 1$ dedicated machines
  - $\mu_{jl} = \mathcal{M}$ parallel machines
  - $\mu_{jl} \subseteq \mathcal{M}$ multipurpose machines

Data that depend on the schedule

- Starting times S_{ij}
- Completion time C_{ij}, C_j

Problem Classification

A scheduling problem is described by a triplet $\alpha \mid \beta \mid \gamma$.

- α machine environment (one or two entries)
- β job characteristics (none or multiple entry)
- γ objective to be minimized (one entry)

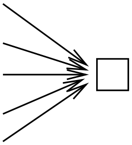
[R.L. Graham, E.L. Lawler, J.K. Lenstra, A.H.G. Rinnooy Kan (1979): Optimization and approximation in deterministic sequencing and scheduling: a survey, Ann. Discrete Math. 4, 287-326.]

$\alpha \mid \beta \mid \gamma$ Classification Scheme

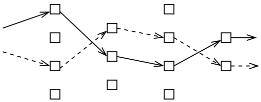
Machine Environment $\qquad \alpha_1\alpha_2 \mid \beta_1 \dots \beta_{13} \mid \gamma$

- single machine/multi-machine ($\alpha_1 = \alpha_2 = 1$ or $\alpha_2 = m$)
- parallel machines: identical ($\alpha_1 = P$), uniform p_j/v_i ($\alpha_1 = Q$), unrelated p_j/v_{ij} ($\alpha_1 = R$)
- multi operations models: Flow Shop ($\alpha_1 = F$), Open Shop ($\alpha_1 = O$), Job Shop ($\alpha_1 = J$), Mixed (or Group) Shop ($\alpha_1 = X$), Multi-processor task sched.

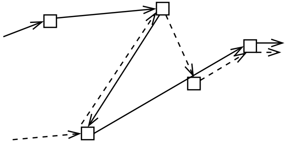
Single Machine



Flexible Flow Shop ($\alpha = FFc$)



Open, Job, Mixed Shop



$\alpha \mid \beta \mid \gamma$ Classification Scheme

Job Characteristics $\alpha_1\alpha_2 \mid \beta_1 \ldots \beta_{13} \mid \gamma$

- $\beta_1 = prmp$ presence of preemption (resume or repeat)
- β_2 precedence constraints between jobs acyclic digraph $G = (V, A)$
  - $\beta_2 = prec$ if G is arbitrary
  - $\beta_2 = \{chains, intree, outtree, tree, sp\text{-}graph\}$
- $\beta_3 = r_j$ presence of release dates
- $\beta_4 = p_j = p$ preprocessing times are equal
- ($\beta_5 = d_j$ presence of deadlines)
- $\beta_6 = \{s\text{-}batch, p\text{-}batch\}$ batching problem
- $\beta_7 = \{s_{jk}, s_{jik}\}$ sequence dependent setup times

$\alpha \mid \beta \mid \gamma$ Classification Scheme

Job Characteristics (2)

$\alpha_1\alpha_2 \mid \beta_1 \ldots \beta_{13} \mid \gamma$

- $\beta_8 = brkdwn$ machine breakdowns
- $\beta_9 = M_j$ machine eligibility restrictions (if $\alpha = Pm$)
- $\beta_{10} = prmu$ permutation flow shop
- $\beta_{11} = block$ presence of blocking in flow shop (limited buffer)
- $\beta_{12} = nwt$ no-wait in flow shop (limited buffer)
- $\beta_{13} = recrc$ recirculation in job shop

$\alpha \mid \beta \mid \gamma$ Classification Scheme

Objective (always $f(C_j)$) $\alpha_1\alpha_2 \mid \beta_1\beta_2\beta_3\beta_4 \mid \gamma$

- Lateness $L_j = C_j - d_j$
- Tardiness $T_j = \max\{C_j - d_j, 0\} = \max\{L_j, 0\}$
- Earliness $E_j = \max\{d_j - C_j, 0\}$
- Unit penalty $U_j = \begin{cases} 1 & \text{if } C_j > d_j \\ 0 & \text{otherwise} \end{cases}$

$\alpha \mid \beta \mid \gamma$ Classification Scheme

Objective $\alpha_1\alpha_2 \mid \beta_1\beta_2\beta_3\beta_4 \mid \gamma$

- Makespan: Maximum completion $C_{max} = \max\{C_1, \ldots, C_n\}$
  tends to max the use of machines
- Maximum lateness $L_{max} = \max\{L_1, \ldots, L_n\}$
- Total completion time $\sum C_j$ (flow time)
- Total weighted completion time $\sum w_j \cdot C_j$
  tends to min the av. num. of jobs in the system, ie, work in progress, or also the throughput time
- Discounted total weighted completion time $\sum w_j(1 - e^{-rC_j})$
- Total weighted tardiness $\sum w_j \cdot T_j$
- Weighted number of tardy jobs $\sum w_j U_j$

All regular functions (nondecreasing in $C_1, \ldots, C_n$) except E_i

$\alpha \mid \beta \mid \gamma$ Classification Scheme

Other Objectives

$\alpha_1\alpha_2 \mid \beta_1\beta_2\beta_3\beta_4 \mid \gamma$

Non regular objectives

- Min $\sum w'_j E_j + \sum w"_j T_j$ (just in time)
- Min waiting times
- Min set up times/costs
- Min transportation costs

RCPS Model

Resource Constrained Project Scheduling Model

Given:

- activities (jobs) $j = 1, \ldots, n$
- renewable resources $i = 1, \ldots, m$
- amount of resources available R_i
- processing times p_j
- amount of resource used r_{ij}
- precedence constraints $j \rightarrow k$

Example