Lecture 2

Solving Problems by Searching

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Last Time

- Agents are used to provide a consistent viewpoint on various topics in the field AI
- Essential concepts:
  - Agents intereact with environment by means of sensors and actuators. A rational agent does "the right thing" $\equiv$ maximizes a performance measure
    ➨ PEAS
  - Environment types: observable, deterministic, episodic, static, discrete, single agent
  - Agent types: table driven (rule based), simple reflex, model-based reflex, goal-based, utility-based, learning agent

Structure of Agents

Agent = Architecture + Program

- Architecture
  - operating platform of the agent
  - computer system, specific hardware, possibly OS
- Program
  - function that implements the mapping from percepts to actions

This course is about the program,
not the architecture

Outline

1. Problem Solving Agents
2. Search
3. Uninformed search algorithms
4. Informed search algorithms
5. Constraint Satisfaction Problem

Outline

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- Problem-solving agents
- Problem types
- Problem formulation
- Example problems
- Basic search algorithms

Problem-solving agents

Restricted form of general agent:

function Simple-Problem-Solving-Agent(*percept*) **returns** an action
  static: *seq*, an action sequence, initially empty
    state, some description of the current world state
    goal, a goal, initially null
    problem, a problem formulation

  state ← Update-State(*state, percept*)
  if *seq* is empty **then**
    goal ← Formulate-Goal(*state*)
    problem ← Formulate-Problem(*state, goal*)
    seq ← Search(*problem*)
  action ← Recommendation(*seq, state*)
  seq ← Remainder(*seq, state*)
  return *action*

Note: this is offline problem solving; solution executed "eyes closed."
Online problem solving involves acting without complete knowledge.

Example: Romania

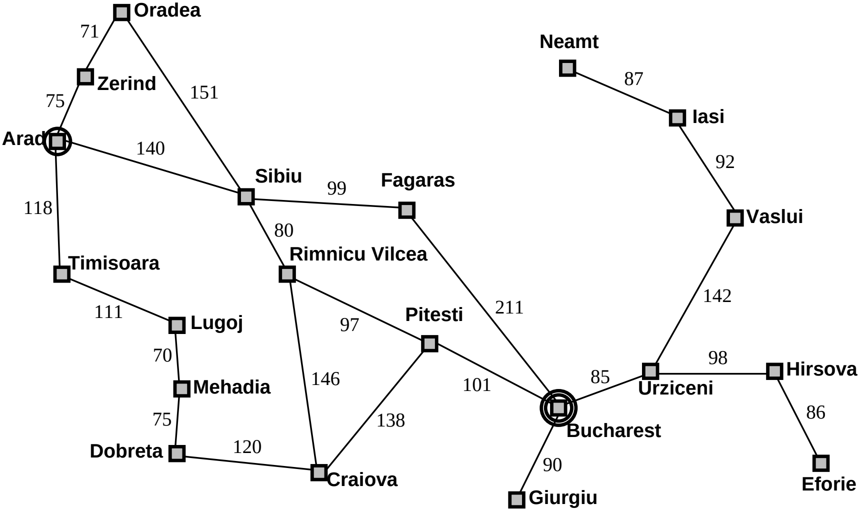
On holiday in Romania; currently in Arad.
Flight leaves tomorrow from Bucharest

Formulate goal:
    be in Bucharest

Formulate problem:
    states: various cities
    actions: drive between cities

Find solution:
    sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest

Example: Romania



State space Problem formulation

A problem is defined by five items:

1. initial state e.g., "at Arad"
2. actions defining the other states, e.g., $Go(Arad)$
3. transition model $res(x, a)$
   e.g., $res(In(Arad), Go(Zerind)) = In(Zerind)$
   alternatively: set of action–state pairs:
   $\{\langle(In(Arad), Go(Zerind)), In(Zerind)\rangle, \ldots\}$
4. goal test, can be
   explicit, e.g., $x =$ "at Bucharest"
   implicit, e.g., $NoDirt(x)$
5. path cost (additive)
   e.g., sum of distances, number of actions executed, etc.
   $c(x, a, y)$ is the step cost, assumed to be ≥ 0

A solution is a sequence of actions
leading from the initial state to a goal state

Selecting a state space

Real world is complex
$\Rightarrow$ state space must be **abstracted** for problem solving

(Abstract) state = set of real states

(Abstract) action = complex combination of real actions
e.g., "Arad $\rightarrow$ Zerind" represents a complex set
of possible routes, detours, rest stops, etc.
For guaranteed realizability, **any** real state "in Arad"
must get to some real state "in Zerind"

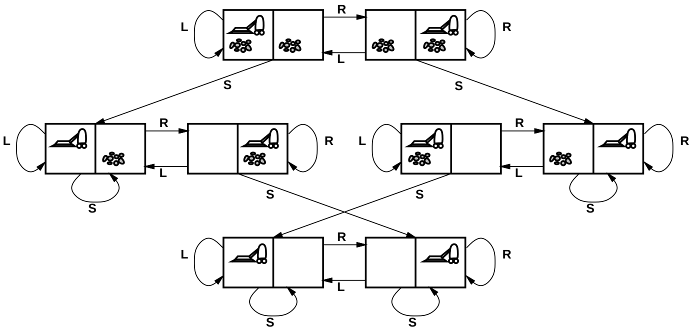
(Abstract) solution =
set of real paths that are solutions in the real world

Each abstract action should be "easier" than the original problem!

Atomic representation

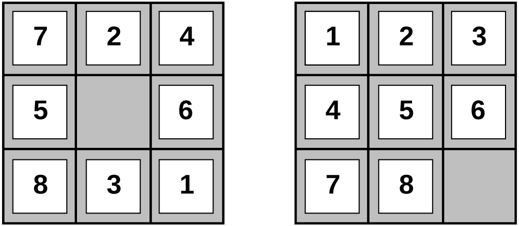
Vacuum world state space graph

Example



states??: integer dirt and robot locations (ignore dirt amounts etc.)
actions??: $Left$, $Right$, $Suck$, $NoOp$
transition model??: arcs in the digraph
goal test??: no dirt
path cost??: 1 per action (0 for $NoOp$)

Example: The 8-puzzle

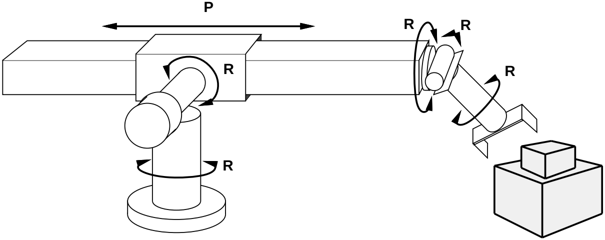


Start State **Goal State**

states??: integer locations of tiles (ignore intermediate positions)
actions??: move blank left, right, up, down (ignore unjamming etc.)
transition model??: effect of the actions
goal test??: = goal state (given)
path cost??: 1 per move

[Note: optimal solution of n-Puzzle family is NP-hard]

Example: robotic assembly



states??: real-valued coordinates of robot joint angles
parts of the object to be assembled

actions??: continuous motions of robot joints

goal test??: complete assembly **with no robot included!**

path cost??: time to execute

Problem types

Deterministic, fully observable, known, discrete $\Longrightarrow$ state space problem
Agent knows exactly which state it will be in; solution is a sequence

Non-observable $\Longrightarrow$ conformant problem
Agent may have no idea where it is; solution (if any) is a sequence

Nondeterministic and/or partially observable $\Longrightarrow$ contingency problem
percepts provide **new** information about current state
solution is a contingent plan or a policy
often **interleave** search, execution

Unknown state space $\Longrightarrow$ exploration problem ("online")

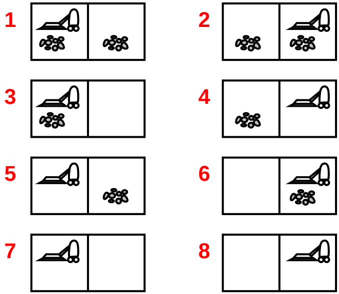
Example: vacuum world

State space, start in #5. Solution??
$[Right, Suck]$

Non-observable, start in $\{1, 2, 3, 4, 5, 6, 7, 8\}$
e.g., $Right$ goes to $\{2, 4, 6, 8\}$. Solution??
$[Right, Suck, Left, Suck]$

Contingency, start in #5
Murphy's Law: $Suck$ can dirty a clean carpet
Local sensing: dirt, location only.
Solution??

$[Right, \textbf{if } dirt \textbf{ then } Suck]$



Example Problems

- Toy problems
  - vacuum cleaner agent
  - 8-puzzle
  - 8-queens
  - cryptarithmetic
  - missionaries and cannibals
- Real-world problems
  - route finding
  - traveling salesperson
  - VLSI layout
  - robot navigation
  - assembly sequencing

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Objectives

- Formulate appropriate problems in optimization and planning (sequence of actions to achive a goal) as search tasks:
  initial state, operators, goal test, path cost
- Know the fundamental search strategies and algorithms
  - uninformed search
    breadth-first, depth-first, uniform-cost, iterative deepening, bi-directional
  - informed search
    best-first (greedy, A*), heuristics, memory-bounded
- Evaluate the suitability of a search strategy for a problem
  - completeness, optimality, time & space complexity

Searching for Solutions

- Traversal of some search space
  from the initial state to a goal state
  legal sequence of actions as defined by operators

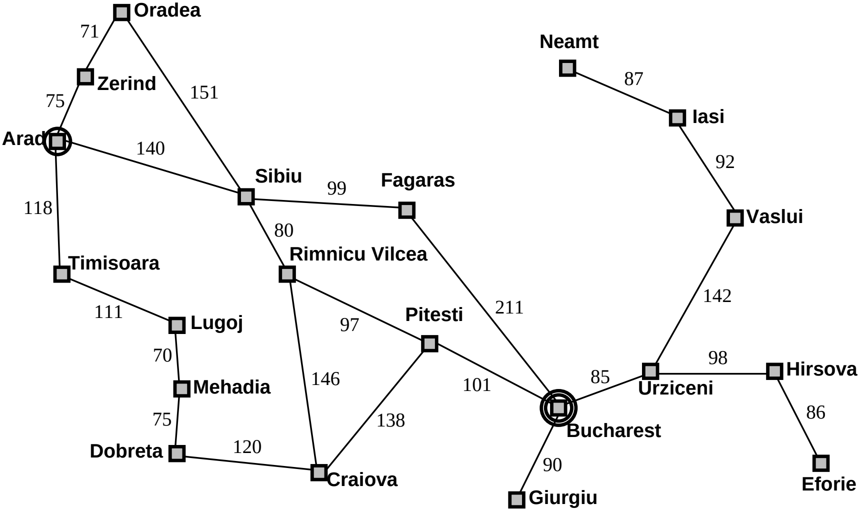
- The search can be performed on

  - On a search tree derived from
    expanding the current state using the possible operators
    Tree-Search algorithm

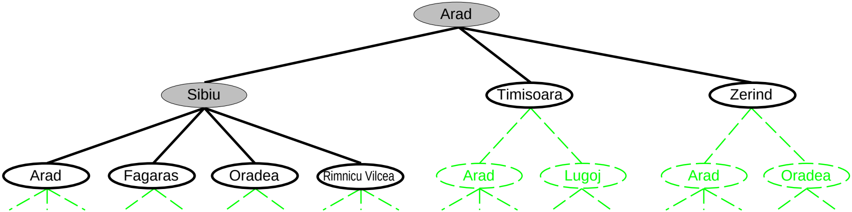
  - A graph representing
    the state space
    Graph-Search algorithm

Search: Terminology

Example: Route Finding



Tree search example



General tree search

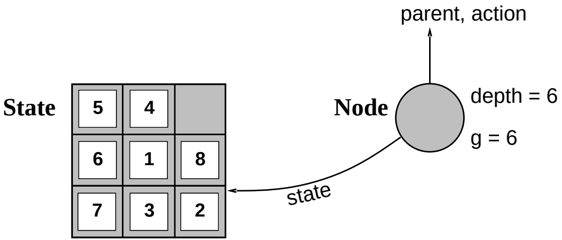
function TREE-SEARCH(*problem*) **returns** a solution, or failure
  initialize the frontier using the initial state of *problem*
  loop do
    if the frontier is empty **then return** failure
    choose a leaf node and remove it from the frontier
    if the node contains a goal state **then return** the corresponding solution
    expand the chosen node, adding the resulting nodes to the frontier

---

function GRAPH-SEARCH(*problem*) **returns** a solution, or failure
  initialize the frontier using the initial state of *problem*
  initialize the explored set to be empty
  loop do
    if the frontier is empty **then return** failure
    choose a leaf node and remove it from the frontier
    if the node contains a goal state **then return** the corresponding solution
    add the node to the explored set
    expand the chosen node, adding the resulting nodes to the frontier
      only if not in the frontier or explored set

Implementation: states vs. nodes

A state is a (representation of) a physical configuration
A node is a data structure constituting part of a search tree
includes state, parent, action, path cost $g(x)$
States do not have parents, children, depth, or path cost!



The Expand function creates new nodes, filling in the various fields using the Transition Model of the problem to create the corresponding states.

Implementation: general tree search

```
function Tree-Search( problem, fringe) returns a solution, or failure
    fringe ← Insert(Make-Node(Initial-State[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← Remove-Front(fringe)
        if Goal-Test(problem, State(node)) then return node
        fringe ← InsertAll(Expand(node, problem), fringe)
```

```
function Expand( node, problem) returns a set of nodes
    successors ← the empty set
    for each action, result in Successor-Fn(problem, State[node]) do
        s ← a new Node
        Parent-Node[s] ← node;   Action[s] ← action;   State[s] ← result
         Path-Cost[s] ← Path-Cost[node] + Step-Cost(State[node], action,
result)
        Depth[s] ← Depth[node] + 1
        add s to successors
    return successors
```

Search strategies

A strategy is defined by picking the **order of node expansion**

```
function Tree-Search( problem, fringe) returns a solution, or failure
   fringe ← Insert(Make-Node(Initial-State[problem]), fringe)
   loop do
       if fringe is empty then return failure
       node ← Remove-Front(fringe)
       if Goal-Test(problem, State(node)) then return node
       fringe ← InsertAll(Expand(node, problem), fringe)
```

Strategies are evaluated along the following dimensions:

- completeness—does it always find a solution if one exists?
- time complexity—number of nodes generated/expanded
- space complexity—maximum number of nodes in memory
- optimality—does it always find a least path cost solution?

Time and space complexity are measured in terms of

- b—maximum branching factor of the search tree
- d—depth of the least-cost solution
- m—maximum depth of the state space (may be ∞)

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Uninformed search strategies

Uninformed strategies use only the information available in the problem definition

Breadth-first search

Uniform-cost search

Depth-first search

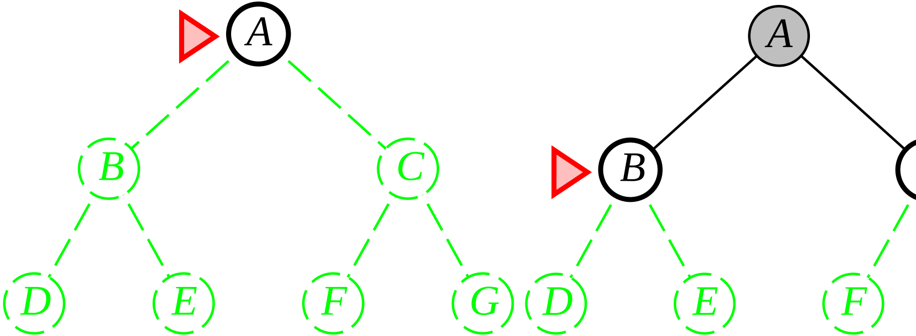
Depth-limited search

Iterative deepening search

Bidirectional Search

Breadth-first search

Expand shallowest unexpanded node (shortest path in the frontier)



Implementation:

fringe is a FIFO queue, i.e., new successors go at end

Properties of breadth-first search

Complete?? Yes (if b is finite)
Optimal?? Yes (if cost = 1 per step); not optimal in general
Time?? $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in d
Space?? $b^{d-1} + b^d = O(b^d)$ (explored + frontier)

Space is the big problem; can easily generate nodes at 100MB/sec
so 24hrs = 8640GB.

Uniform-cost search

Expand first least-cost path

(Equivalent to breadth-first if step costs all equal)

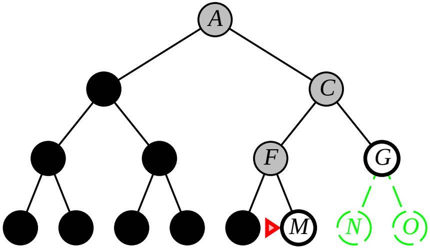
Implementation:

fringe = priority queue ordered by path cost, lowest first,

Complete?? Yes, if step cost $\geq \epsilon$
Optimal??Yes—nodes expanded in increasing order of $g(n)$
Time?? # of nodes with $g \leq$ cost of optimal solution, $O(b^{1+\lceil C^*/\epsilon \rceil})$
where C^* is the cost of the optimal solution
Space??# of nodes with $g \leq$ cost of optimal solution, $O(b^{1+\lceil C^*/\epsilon \rceil})$

Depth-first search

Expand deepest unexpanded node



Implementation:

fringe = LIFO queue, i.e., put successors at front

Properties of depth-first search

Complete?? No: fails in infinite-depth spaces, or spaces with loops
Modify to avoid repeated states along path
$\Rightarrow$ complete in finite spaces
Optimal?? No
Time?? $O(b^m)$: terrible if m is much larger than d
but if solutions are dense, may be much faster than breadth-first
Space?? $O(bm)$, i.e., linear space!

Depth-limited search

= depth-first search with depth limit l,
i.e., nodes at depth l have no successors
Recursive implementation:

function Depth-Limited-Search(*problem*, *limit*) **returns** soln/fail/cutoff
    Recursive-DLS(Make-Node(Initial-State[*problem*]), *problem*, *limit*)

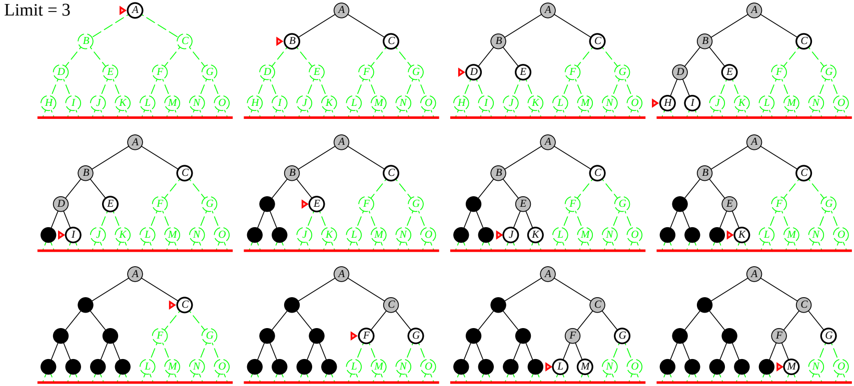
function Recursive-DLS(*node*, *problem*, *limit*) **returns** soln/fail/cutoff
    cutoff-occurred? ← false
    if Goal-Test(*problem*, State[*node*]) **then return** *node*
    else if Depth[*node*] = *limit* **then return** *cutoff*
    else for each *successor* **in** Expand(*node*, *problem*) **do**
        result ← Recursive-DLS(*successor*, *problem*, *limit*)
        if *result* = *cutoff* **then** *cutoff-occurred?* ← true
        else if *result* ≠ *failure* **then return** *result*
    if *cutoff-occurred?* **then return** *cutoff* **else return** *failure*

Iterative deepening search

```
function Iterative-Deepening-Search( problem) returns a solution
    inputs: problem, a problem

    for depth ← 0 to ∞ do
        result ← Depth-Limited-Search( problem, depth)
        if result ≠ cutoff then return result
    end
```

Iterative deepening search



Properties of iterative deepening search

Complete?? Yes
Optimal?? Yes, if step cost = 1
Can be modified to explore uniform-cost tree
Time?? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$
Space?? $O(bd)$

Numerical comparison in time for $b = 10$ and $d = 5$, solution at far right leaf:

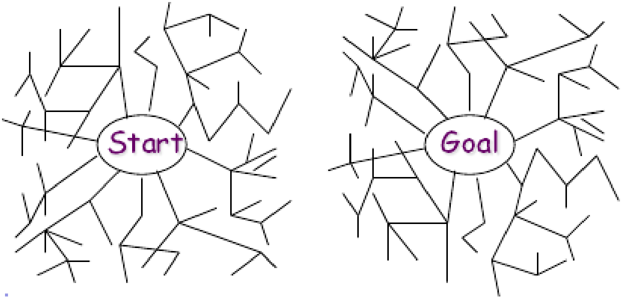
$$
\begin{aligned}
N\text{IDS}) &= 50 + 400 + 3,000 + 20,000 + 100,000 = 123,450 \\
N(\text{BFS}) &= 10 + 100 + 1,000 + 10,000 + 100,000 + 999,990 = 1,111,100
\end{aligned}
$$

IDS does better because other nodes at depth d are not expanded
BFS can be modified to apply goal test when a node is **generated**
Iterative lenghtening not as successul as IDS

Bidirectional Search

- Search simultaneously (using breadth-first search)
  from goal to start
  from start to goal

- Stop when the two search trees intersects



Difficulties in Bidirectional Search

- If applicable, may lead to substantial savings
- Predecessors of a (goal) state must be generated
  Not always possible, eg. when we do not know the optimal state explicitly
- Search must be coordinated between the two search processes.
- What if many goal states?
- One search must keep all nodes in memory

Summary of algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes*	Yes*	No	Yes, if $l \geq d$	Yes
Time	b^{d+1}	$b^{\lceil 1+C^*/\epsilon \rceil}$	b^m	b^l	b^d
Space	b^{d+1}	$b^{\lceil 1+C^*/\epsilon \rceil}$	bm	bl	bd
Optimal?	Yes*	Yes	No	No	Yes*

Summary

- Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored
- Variety of uninformed search strategies
- Iterative deepening search uses only linear space and not much more time than other uninformed algorithms
- Graph search can be exponentially more efficient than tree search

Outline

Review: Tree search

```
function Tree-Search( problem, fringe) returns a solution, or failure
    fringe ← Insert(Make-Node(Initial-State[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← Remove-Front(fringe)
        if Goal-Test[problem] applied to State(node) succeeds return node
        fringe ← InsertAll(Expand(node, problem), fringe)
```

A strategy is defined by picking the **order of node expansion**

Informed search strategy

Informed strategies use agent's background information about the problem
map, costs of actions, approximation of solutions, ...

- best-first search
  - greedy search
  - A*search
- local search (not in this course)
  - Hill-climbing
  - Simulated annealing
  - Genetic algorithms
  - Local search in continuous spaces

Best-first search

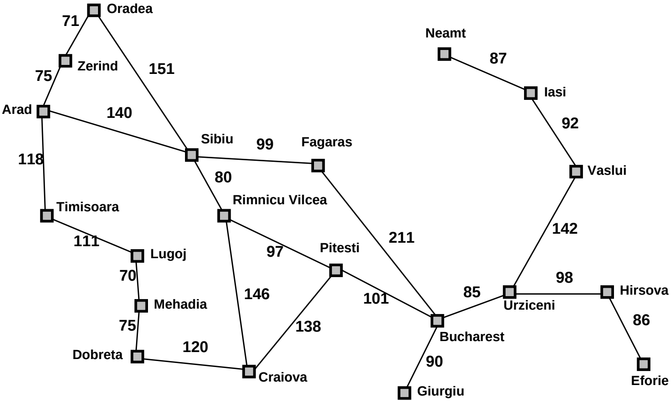
Idea: use an evaluation function for each node
– estimate of "desirability"

⇒ Expand most desirable unexpanded node

Implementation:
fringe is a queue sorted in decreasing order of desirability

Special cases:
greedy search
A* search

Romania with step costs in km



Straight−line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

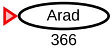
Greedy search

Evaluation function $h(n)$ (**h**euristic)
$\quad = $ estimate of cost from n to the closest goal

E.g., $h_{\mathrm{SLD}}(n)$ = straight-line distance from n to Bucharest

Greedy search expands the node that **appears** to be closest to goal

Greedy search example



Properties of greedy search

Complete?? No–can get stuck in loops, e.g., from Iasi to Fargas

Iasi $\rightarrow$ Neamt $\rightarrow$ Iasi $\rightarrow$ Neamt $\rightarrow$

Complete in finite space with repeated-state checking

Optimal?? No

Time?? $O(b^m)$, but a good heuristic can give dramatic improvement

Space?? $O(b^m)$

A* search

Idea: avoid expanding paths that are already expensive
Evaluation function $f(n) = g(n) + h(n)$

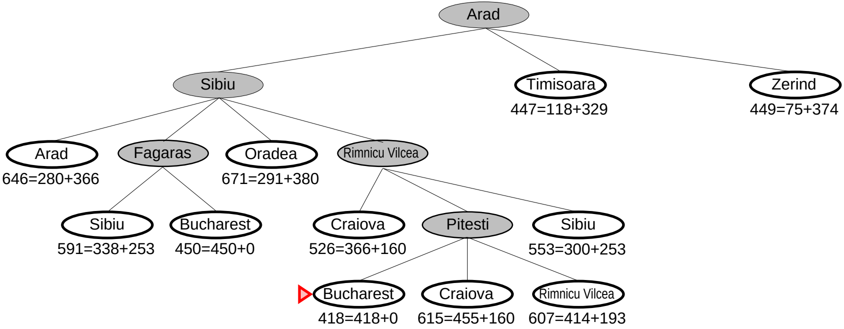
$g(n)$ = cost so far to reach n
$h(n)$ = estimated cost to goal from n
$f(n)$ = estimated total cost of path through n to goal

A* search uses an admissible heuristic
i.e., $h(n) \leq h^*(n)$ where $h^*(n)$ is the **true** cost from n.
(Also require $h(n) \geq 0$, so $h(G) = 0$ for any goal G.)

E.g., $h_{\mathrm{SLD}}(n)$ never overestimates the actual road distance

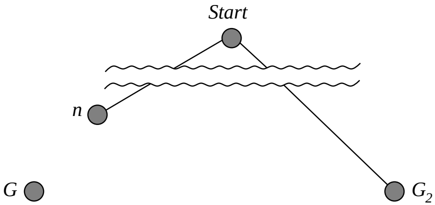
Theorem: A* search is optimal

A* search example



Optimality of A* (standard proof)

Suppose some suboptimal goal G_2 has been generated and is in the queue. Let n be an unexpanded node on a shortest path to an optimal goal G_1.

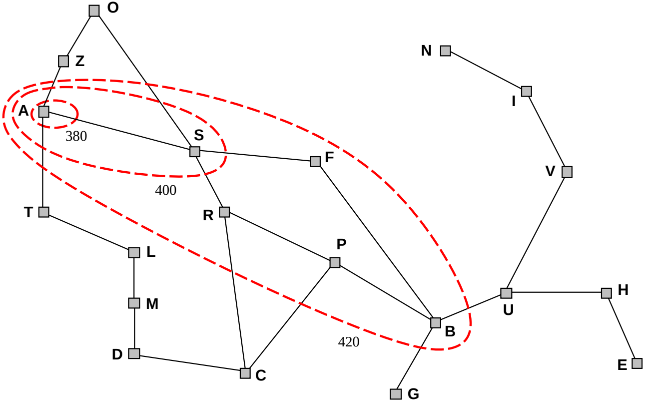


$$
\begin{aligned}
f(G_2) &= g(G_2) && \text{since } h(G_2) = 0 \\
&> g(G_1) && \text{since } G_2 \text{ is suboptimal} \\
&\geq f(n) && \text{since } h \text{ is admissible}
\end{aligned}
$$

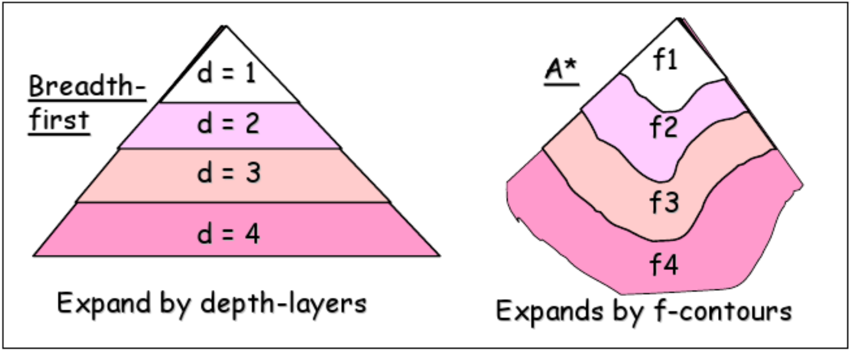
Since $f(G_2) > f(n)$, A* will never select G_2 for expansion

Optimality of A* (more useful)

Lemma: A* expands nodes in order of increasing f value*
Gradually adds "f-contours" of nodes (cf. breadth-first adds layers)
Contour i has all nodes with $f = f_i$, where $f_i < f_{i+1}$



Astar vs. Depth search



Properties of A*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$
Optimal?? Yes—cannot expand f_{i+1} until f_i is finished
A* expands all nodes with $f(n) < C^*$
A* expands some nodes with $f(n) = C^*$
A* expands no nodes with $f(n) > C^*$
Time?? Exponential in [relative error in $h \times$ length of sol.]
Space?? Keeps all nodes in memory

Proof of lemma: Consistency

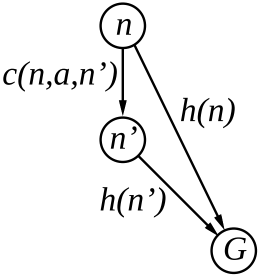
A heuristic is consistent if

$$h(n) \leq c(n, a, n') + h(n')$$

If h is consistent, we have

$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n, a, n') + h(n') \\ &\geq g(n) + h(n) \\ &= f(n) \end{aligned}$$

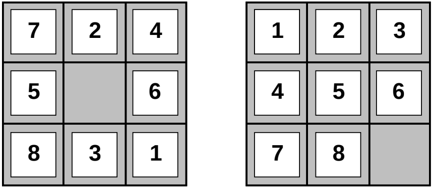
I.e., $f(n)$ is nondecreasing along any path.



Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)



Start State   Goal State

$\underline{h_1(S)\ =}$??
$\underline{h_2(S)\ =}$??

Admissible heuristics

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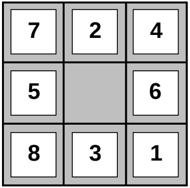
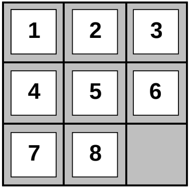


Start State



Goal State

$h_1(S) =$?? 6
$h_2(S) =$?? 4+0+3+3+1+0+2+1 = 14

Dominance

If $h_2(n) \geq h_1(n)$ for all n (both admissible)
then h_2 dominates h_1 and is better for search

Typical search costs:

$d = 14$	IDS = 3,473,941 nodes
	A$^*(h_1)$ = 539 nodes
	A$^*(h_2)$ = 113 nodes
$d = 24$	IDS $\approx$ 54,000,000,000 nodes
	A$^*(h_1)$ = 39,135 nodes
	A$^*(h_2)$ = 1,641 nodes

Given any admissible heuristics h_a, h_b,

$$h(n) = \max(h_a(n), h_b(n))$$

is also admissible and dominates h_a, h_b

Relaxed problems

Admissible heuristics can be derived from the **exact** solution cost of a **relaxed** version of the problem

- If the rules of the 8-puzzle are relaxed so that a tile can move **anywhere**, then $h_1(n)$ gives the shortest solution
- If the rules are relaxed so that a tile can move to **any adjacent square**, then $h_2(n)$ gives the shortest solution
- Key point: the optimal solution cost of a relaxed problem is no greater than the optimal solution cost of the real problem

Memory-Bounded Heuristic Search

- Try to reduce memory needs
- Take advantage of heuristic to improve performance
  - Iterative-deepening A*(IDA*)
  - SMA*

Iterative Deepening A*

- Uniformed Iterative Deepening (repetition)
  - depth-first search where the max depth is iteratively increased
- IDA*
  - depth-first search, but only nodes with f-cost less than or equal to smallest f-cost of nodes expanded at last iteration
  - was the "best" search algorithm for many practical problems

Properties of IDA*

Complete?? Yes
Time complexity?? Still exponential
Space complexity?? linear
Optimal?? Yes. Also optimal in the absence of monotonicity

Simple Memory-Bounded A*

Use all available memory

- Follow A*algorithm and fill memory with new expanded nodes
- If new node does not fit
  - remove stored node with worst f-value
  - propagate f-value of removed node to parent
- SMA*will regenerate a subtree only when it is needed
  the path through subtree is unknown, but cost is known

Propeties of SMA*

Complete?? yes, if there is enough memory for the shortest solution path
Time?? same as A*if enough memory to store the tree
Space?? use available memory
Optimal?? yes, if enough memory to store the best solution path

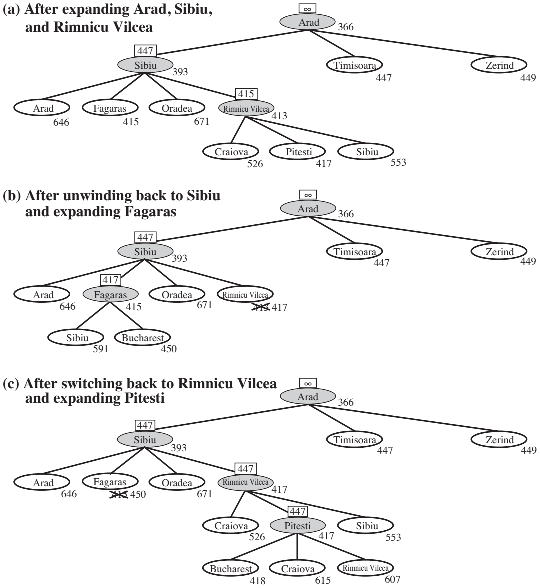
In practice, often better than A*and IDA*trade-off between time and space requirements

Recursive Best First Search

```
function RECURSIVE-BEST-FIRST-SEARCH(problem) returns a solution, or failure
   return RBFS(problem, MAKE-NODE(problem.INITIAL-STATE), ∞)

function RBFS(problem, node, f_limit) returns a solution, or failure and a new f-cost limit
  if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
  successors ← [ ]
  for each action in problem.ACTIONS(node.STATE) do
     add CHILD-NODE(problem, node, action) into successors
  if successors is empty then return failure, ∞
  for each s in successors do /* update f with value from previous search, if any */
     s.f ← max(s.g + s.h, node.f))
  loop do
     best ← the lowest f-value node in successors
     if best.f > f_limit then return failure, best.f
     alternative ← the second-lowest f-value among successors
     result, best.f ← RBFS(problem, best, min(f_limit, alternative))
     if result ≠ failure then return result
```

Recursive Best First Search



Outline

1. Problem Solving Agents
2. Search
3. Uninformed search algorithms
4. Informed search algorithms
5. Constraint Satisfaction Problem

Constraint Satisfaction Problem (CSP)

Standard search problem:
state is a "black box"—any old data structure
that supports goal test, eval, successor

CSP:
state is defined by variables X_i with values from domain D_i

goal test is a set of constraints specifying
allowable combinations of values for subsets of variables

Simple example of a **formal representation language**

Allows useful **general-purpose** algorithms with more power than standard search algorithms

Standard search formulation

States are defined by the values assigned so far

- ♢ Initial state: the empty assignment, $\{\,\}$
- ♢ Successor function: assign a value to an unassigned variable that does not conflict with current assignment.
  $\Longrightarrow$ fail if no legal assignments (not fixable!)
- ♢ Goal test: the current assignment is complete

1) This is the same for all CSPs! 😈
2) Every solution appears at depth n with n variables
   $\Longrightarrow$ use depth-first search
3) Path is irrelevant, so can also use complete-state formulation
4) $b = (n - \ell)d$ at depth ℓ, hence $n!d^n$ leaves!!!! ☹

Backtracking search

Variable assignments are commutative, i.e.,
$[WA = red$ then $NT = green]$ same as
$[NT = green$ then $WA = red]$
Only need to consider assignments to a single variable at each node
$\implies b = d$ and there are d^n leaves

Depth-first search for CSPs with single-variable assignments
is called backtracking search
Backtracking search is the basic uninformed algorithm for CSPs
Can solve n-queens for $n \approx 25$

Backtracking search

```
function Backtracking-Search(csp) returns solution/failure
    return Recursive-Backtracking({ }, csp)

function Recursive-Backtracking(assignment, csp) returns soln/failure
    if assignment is complete then return assignment
    var ← Select-Unassigned-Variable(Variables[csp], assignment, csp)
    for each value in Order-Domain-Values(var, assignment, csp) do
        if value is consistent with assignment given Constraints[csp] then
            add {var = value} to assignment
            result ← Recursive-Backtracking(assignment, csp)
            if result ≠ failure then return result
            remove {var = value} from assignment
    return failure
```

Summary

Uninformed Search

- Breadth-first search
- Uniform-cost search
- Depth-first search
- Depth-limited search
- Iterative deepening search
- Bidirectional Search

Informed Search

- best-first search
  - greedy search
  - A*search
  - Iterative Deepening A*
  - Memory bounded A*
  - Recursive best first

Constraint Satisfaction and Backtracking