DM204 – Autumn 2013

Scheduling, Timetabling and Routing

Lecture 9
More on Transportation Scheduling

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Outline

1. Vehicle Scheduling

2. Transport Timetabling
   - Train Timetabling

Course Overview

- ✔ Scheduling
  - ✔ Classification
  - ✔ Complexity issues
  - ✔ Single Machine
  - Parallel Machine
  - Flow Shop and Job Shop
  - Resource Constrained Project Scheduling Model

- Timetabling
  - Sport Timetabling
  - Reservations and Education
  - University Timetabling
  - ✔ Public Transports
  - ✔ Crew Scheduling

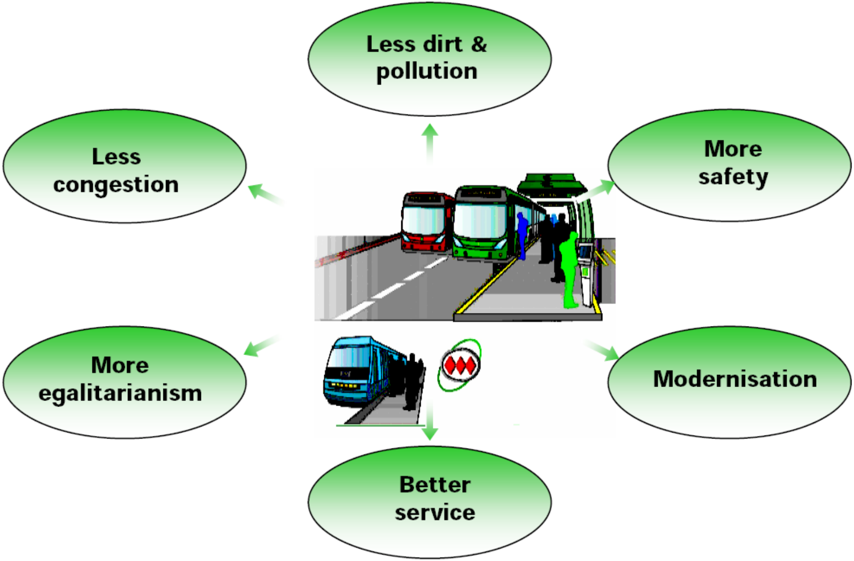
- Vechicle Routing
  - Capacited Models
  - Time Windows models
  - Rich Models

Planning problems in public transport

Phase:	Planning	Scheduling	Dispatching
Horizon:	Long Term	Timetable Period	Day of Operation
Objective:	Service Level	Cost Reduction	Get it Done
Steps:	Network Design Line Planning Timetabling Fare Planning	Vehicle Scheduling Duty Scheduling Duty Rostering	Crew Assignment Delay Management Failure Management Depot Management

↑ Master Schedule —Dynamic Management→ Conflict resolution ↑

[Borndörfer, Grötschel, Pfetsch, 2005, ZIB-Report 05-22]



[Borndörfer, Liebchen, Pfetsch, course 2006, TU Berlin]

Outline

1. Vehicle Scheduling

2. Transport Timetabling
    - Train Timetabling

OR in Transports

We consider here:

- Vehicle (Truck/Buses) Routing (seen last week)
- Tanker Scheduling
- Daily Aricraft Scheduling

Tanker Scheduling
(sec. 11.2)

Input:

- p ports

  limits on the physical characteristics of the ships

- n cargoes:

  type, quantity, load port, delivery port, time window constraints on the load and delivery times

- ships (tanker): s company-owned plus others chartered
  Each ship has a capacity, draught, speed, fuel consumption, starting location and times

  These determine the costs of a shipment: c_i^l (company-owned) c_j^* (chartered)

Output: A schedule for each ship, that is, an itinerary listing the ports visited and the time of entry in each port within the rolling horizon such that the total cost of transportation is minimized

Two phase approach:

1. determine for each ship i the set S_i of all possible itineraries
2. select the itineraries for the ships by solving an IP problem

Phase 1 can be solved by some ad-hoc enumeration or heuristic algorithm that checks the feasibility of the itinerary and its cost.
Phase 2 Set packing problem with additional constraints (next slide)

For each itinerary l of ship i compute the profit with respect to charter:

$$\pi_i^l = \sum_{j=1}^{n} a_{ij}^l c_j^* - c_i^l$$

where $a_{ij}^l = 1$ if cargo j is shipped by ship i in itinerary l and 0 otherwise.

A set packing model with additional constraints

Variables

$$x_i^l \in \{0,1\} \qquad \forall i = 1,\ldots,s;\ l \in S_i$$

Each cargo is assigned to at most one ship:

$$\sum_{i=1}^{s} \sum_{l \in S_i} a_{ij}^l x_i^l \leq 1 \qquad \forall j = 1,\ldots,n$$

Each tanker can be assigned at most one itinerary

$$\sum_{l \in S_i} x_i^l \leq 1 \qquad \forall i = 1,\ldots,s$$

Objective: maximize profit

$$\max \sum_{i=1}^{s} \sum_{l \in S_i} \pi_i^l x_i^l$$

Customized branching mechanisms

- select variable x_i^l and branch with $x_i^l = 0$ and $x_i^l = 1$
  select variable with value closest to 0.5
  for $x_i^l = 1$ remove schedules for other ships that have cargo in common

- select a ship i and generate for each schedule l in S_i a branch with $x_i^l = 1$
  select the ship for example on the basis of its importance, or select the one with most fractional number

OR in Air Transport Industry

- Aircraft and Crew Schedule Planning
  - Schedule Design (specifies legs and times)
  - Fleet Assignment
  - Aircraft Maintenance Routing
  - Crew Scheduling
    - crew pairing problem
    - crew assignment problem (bidlines)
- Airline Revenue Management
  - number of seats available at fare level
  - overbooking
  - fare class mix (nested booking limits)
- Aviation Infrastructure
  - airports
    - runaways scheduling (queue models, simulation; dispatching, optimization)
    - gate assignments
  - air traffic management

Daily Aircraft Routing and Scheduling (Sec. 11.3)

[Desaulniers, Desrosiers, Dumas, Solomon and Soumis, 1997]

Input:

- L set of flight legs with airport of origin and arrival, departure time windows $[e_i, l_i]$, $i \in L$, duration, cost/revenue
- Heterogeneous aircraft fleet T, with m_t aircrafts of type $t \in T$

Output: For each aircraft, a sequence of operational flight legs and departure times such that operational constraints are satisfied:

- number of planes for each type
- restrictions on certain aircraft types at certain times and certain airports
- required connections between flight legs ("thrus")
- limits on daily traffic at certain airports
- balance of airplane types at each airport

and the total profits are maximized.

- L_t denotes the set of flights that can be flown by aircraft of type t
- S_t the set of feasible schedules for an aircraft of type t (inclusive of the empty set)
- $a^l_{ti} = \{0, 1\}$ indicates if leg i is covered by $l \in S_t$
- π_{ti} profit of covering leg i with aircraft of type i

$$\pi^l_t = \sum_{i \in L_t} \pi_{ti} a^l_{ti} \qquad \text{for } l \in S_t$$

- P set of airports, P_t set of airports that can accommodate type t
- o^l_{tp} and d^l_{tp} equal to 1 if schedule l, $l \in S_t$ starts and ends, resp., at airport p

A set partitioning model with additional constraints

Variables

$$x_t^l \in \{0,1\} \qquad \forall t \in T;\ l \in S_t \qquad \text{and} \qquad x_t^0 \in \mathbf{N} \qquad \forall t \in T$$

Maximum number of aircraft of each type:

$$\sum_{l \in S_t} x_t^l = m_t \qquad \forall t \in T$$

Each flight leg is covered exactly once:

$$\sum_{t \in T} \sum_{l \in S_t} a_{ti}^l x_t^l = 1 \qquad \forall i \in L$$

Flow conservation at the beginning and end of day for each aircraft type

$$\sum_{l \in S_t} (o_{tp}^l - d_{tp}^l) x_t^l = 0 \qquad \forall t \in T;\ p \in P$$

Maximize total anticipated profit

$$\max \sum_{t \in T} \sum_{l \in S_t} \pi_t^l x_t^l$$

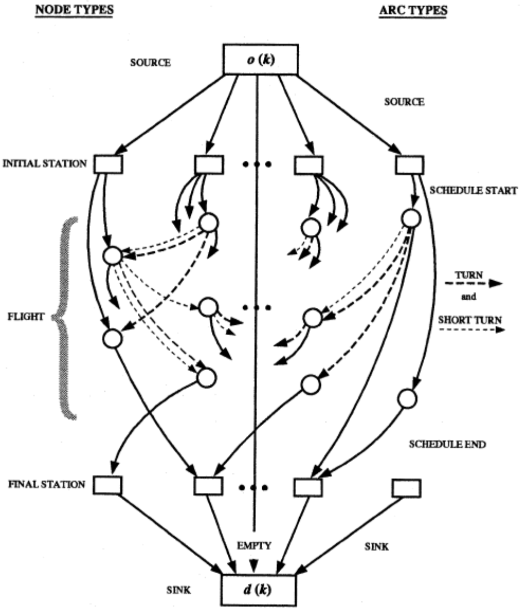
Solution Strategy: branch-and-price

- At the high level branch-and-bound similar to the Tanker Scheduling case
- Upper bounds obtained solving linear relaxations by column generation.
  - Decomposition into
    - Restricted Master problem, defined over a restricted number of schedules
    - Subproblem, used to test the optimality or to find a new feasible schedule to add to the master problem (column generation)
  - Each restricted master problem solved by LP.
    It finds current optimal solution and dual variables
  - Subproblem (or pricing problem) corresponds to finding longest path with time windows in a network defined by using dual variables of the current optimal solution of the master problem. Solved by dynamic programming.

source arcs
sink arcs
schedule origination arcs
schedule termination arcs
turn arcs

Potential profit:

$$\bar{\pi}_i^l = \sum_{i \in L_t} (\pi_{ti} - \alpha_i) a_{ti}^l - \beta_t - \sum_{p \in P_t} \gamma_{tp} (d_{tp}^l - o_t^l$$



$$\text{Maximize} \sum_{k \in K} \sum_{(i,j) \in A^k} c_{ij}^k X_{ij}^k \tag{8}$$

subject to:

$$\sum_{k \in K} \sum_{j:(i,j) \in A^k} X_{ij}^k = 1 \quad \forall i \in N, \tag{9}$$

$$\sum_{i:(i,s) \in NS_2^k} X_{is}^k - \sum_{j:(s,j) \in S_1 N^k} X_{sj}^k = 0 \quad \forall k \in K, \forall s \in S^k, \tag{10}$$

$$\sum_{s \in S_1^k} X_{o(k),s}^k + X_{o(k),d(k)}^k = n^k \quad \forall k \in K, \tag{11}$$

$$\sum_{i:(i,j) \in A^k} X_{ij}^k - \sum_{i:(j,i) \in A^k} X_{ji}^k = 0$$

$$\forall k \in K, \forall j \in V^k \setminus \{o(k), d(k)\}, \tag{12}$$

$$\sum_{s \in S_2^k} X_{s,d(k)}^k + X_{o(k),d(k)}^k = n^k \quad \forall k \in K, \tag{13}$$

$$X_{ij}^k \geq 0 \quad \forall k \in K, \forall (i,j) \in A^k, \tag{14}$$

$$a_i^k \leq T_i^k \leq b_i^k \quad \forall k \in K, \forall i \in V^k, \tag{15}$$

$$X_{ij}^k (T_i^k + d_{ij}^k - T_j^k) \leq 0 \quad \forall k \in K, \forall (i,j) \in A^k, \tag{16}$$

$$X_{ij}^k \text{ integer} \quad \forall k \in K, \forall (i,j) \in A^k. \tag{17}$$

B&B strategies

- 0–1 branching on set partitioning formulation:
  1 leads to removal of flights covered from the network
  but 0 is more complicated to handle: paths forbidden in the pricing problem

- multicommodity formulation
  can decompose back from node-path to node-arc
  - binary branching forced on the flow variables x_{ij}^k variables
  - on linear combination of flow variables such as:

    $$X_{ij} = \sum_{k \in K} X_{ij}^k \quad \forall i, j \in N^k$$

    ie, connection ij to 0–1 whatever is the aircraft
  - or

    $$X_i^k = \sum_{ij \in A^k} X_{ij}^k \quad \forall k \in K, i \in N^k$$

    ie, assignment of aircraft of type k to flight leg i to 0–1
  - or number of aircraft of type k used:

    $$X_{\sigma(k)}^k = \sum_{s \in S_i^k} X_{\sigma(k),s}^k \quad \forall k \in K$$

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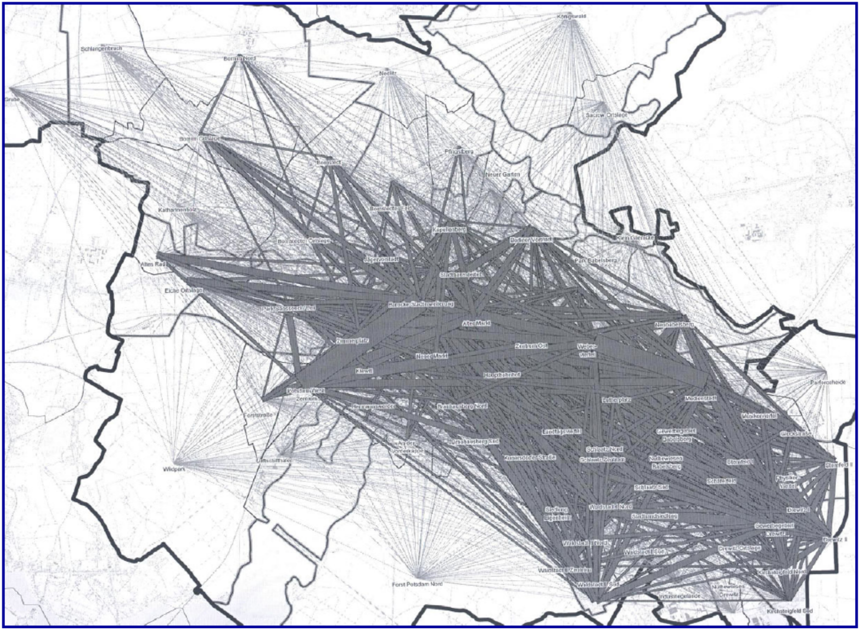
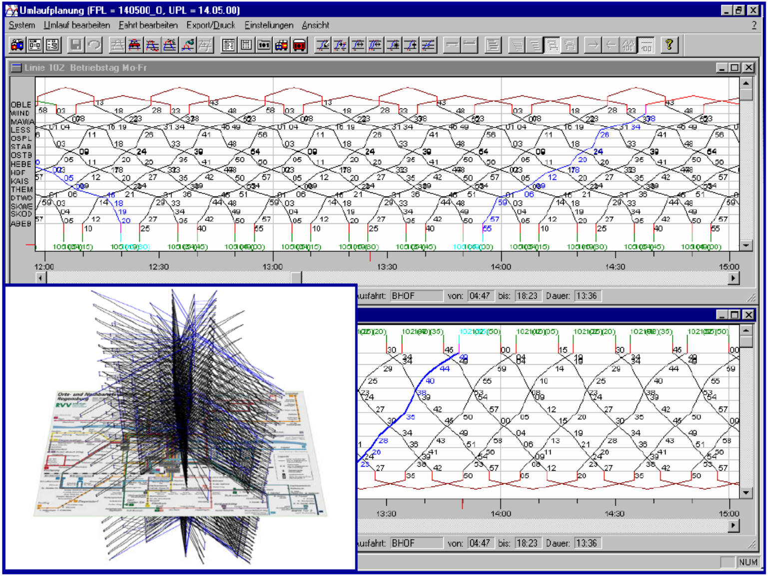
Line Planning



Time-space diagram



[Borndörfer, Liebchen, Pfetsch, course 2006, TU Berlin]

Terminology

- Rolling stock, block, freight or passenger trains
- Railway network, track
- Stations, junctions
- Headway
- Dwell time and transfer/correspondence times
- Line, Line planning problem
- Train timetabling problem; integrated, symmetric, periodic timetable; robustness
- Train routing/platforming problem
- Train sequencing/pathing/scheduling
- Train dispatching/rescheduling problem
- Shunting/parking problem
- Marshalling/classification problem

Here we see:

- Train timetabling
    - single track
    - perodic railways timetabling model (PESP)
- Train dispatching
- Crew scheduling (already seen)

Train Timetabling

Input:

- Corridors made up of two independent one-way tracks
- L links between $L+1$ stations.
- T set of trains and T_j, $T_j \subseteq T$, subset of trains that pass through link j

Output: We want to find a periodic (eg, one day) timetable for the trains on one track (the other can be mirrored) that specifies:

- y_{ij} = time train i enters link j
- z_{ij} = time train i exists link j

such that specific constraints are satisfied and costs minimized.

Constraints:

- Minimal time to traverse one link
- Minimum stopping times at stations to allow boarding
- Minimum headways between consecutive trains on each link for safety reasons
- Trains can overtake only at train stations
- There are some "*predetermined*" upper and lower bounds on arrival and departure times for certain trains at certain stations

Costs due to:

- deviations from some "*preferred*" arrival and departure times for certain trains at certain stations
- deviations of the travel time of train i on link j
- deviations of the dwelling time of train i at station j

Solution Approach

- All constraints and costs can be modeled in a MIP with the variables: y_{ij}, z_{ij} and $x_{ihj} = \{0, 1\}$ indicating if train i precedes train h
- Two dummy trains T' and T'' with fixed times are included to compact and make periodic
- Large model solved heuristically by decomposition.
- Key Idea: insert one train at a time and solve a simplified MIP.
- In the simplified MIP the order in each link of trains already scheduled is maintained fixed while times are recomputed. The only order not fixed is the one of the new train inserted k (x_{ihj} simplifies to x_{ij} which is 1 if k is inserted in j after train i)

Overall Algorithm

Step 1 (Initialization)
Introduce in T_0 two "dummy trains" as first and last trains

Step 2 (Select an Unscheduled Train) Select the next train k through the train selection priority rule

Step 3 (Set up and preprocess the MIP) Include train k in set T_0
Set up MIP(K) for the selected train k
Preprocess MIP(K) to reduce number of 0–1 variables and constraints

Step 4 (Solve the MIP) Solve MIP(k). If algorithm does not yield feasible solution STOP.
Otherwise, add train k to the list of already scheduled trains and fix for each link the sequences of all trains in T_0.

Step 5 (Reschedule all trains scheduled earlier) Consider the current partial schedule that includes train k.
For each train $i \in \{T_0 - k\}$ delete it and reschedule it

Step 6 (Stopping criterion) If T_0 consists of all train, then STOP otherwise go to Step 2.

PESP model

Periodic event scheduling problem:

in terms of potentials

$$
\begin{aligned}
\min\ & w_{ij}^T(\pi_j - \pi_i + T \cdot p_{ij} - l_{ij}) \\
& l_{ij} \leq \pi_j - \pi_i + T \cdot p_{ij} \leq u_{ij} \quad \forall ij \in A \\
& 0 \leq \pi_i \leq T(1-\epsilon) \quad \forall i \in V \\
& p \in Z^{|A|}
\end{aligned}
$$

in terms tensions $x_a = \pi_j - \pi_i + T \cdot p_a$

$$
\begin{aligned}
\min\ & w^T x \\
& \sum_{a \in C^+} x_a - \sum_{a \in C^-} x_a = Tq_C \quad \forall C \in \mathcal{C} \\
& l_a \leq x_a \leq u_a \quad \forall a \in A \\
& q_c \in Z, \quad \forall C \in \mathcal{C}
\end{aligned}
$$