DM811
Heuristics for Combinatorial Optimization

Examples

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Examples

Iterative Improvement for TSP

TSP-2opt-first(s)
input: *an initial candidate tour* $s \in S(\in)$
output: *a local optimum* $s \in S_\pi$

for $i = 1$ to $n - 1$ **do**
  for $j = i + 1$ to n **do**
    if $P[i] + 1 = P[j]$ or $P[j] + 1 = P[i]$ **then** *continue*
    if $P[i] + 1 \geq n$ or $P[j] + 1 \geq n$ **then** *continue*

$$\begin{aligned}\Delta_{ij} \quad = \quad & d(\pi_{P[i]}, \pi_{P[j]}) + d(\pi_{P[i]+1}, \pi_{P[j]+1}) + \\ & -d(\pi_{P[i]}, \pi_{P[i]+1}) - d(\pi_{P[j]}, \pi_{P[j]+1})\end{aligned}$$

    if $\Delta_{ij} < 0$ **then**
      $\mathtt{UpdateTour(s, i, j)}$

is it really?

Examples

Iterative Improvement for TSP

TSP-2opt-first(s)
input: *an initial candidate tour* $s \in S(\in)$
output: *a local optimum* $s \in S_\pi$

Improvement:=$\mathtt{TRUE}$;
while Improvement is `TRUE` **do**
  Improvement:=$\mathtt{FALSE}$;
  for $i = 1$ to $n - 1$ **do**
    for $j = i + 1$ to n **do**
      if $P[i] + 1 = P[j]$ or $P[j] + 1 = P[i]$ **then** *continue*
      if $P[i] + 1 \geq n$ or $P[j] + 1 \geq n$ **then** *continue*

$$\begin{aligned}\Delta_{ij} \quad = \quad & d(\pi_{P[i]}, \pi_{P[j]}) + d(\pi_{P[i]+1}, \pi_{P[j]+1}) + \\ & -d(\pi_{P[i]}, \pi_{P[i]+1}) - d(\pi_{P[j]}, \pi_{P[j]+1})\end{aligned}$$

      if $\Delta_{ij} < 0$ **then**
        $\mathtt{UpdateTour}(\mathtt{s}, \mathtt{i}, \mathtt{j})$
        Improvement=$\mathtt{TRUE}$

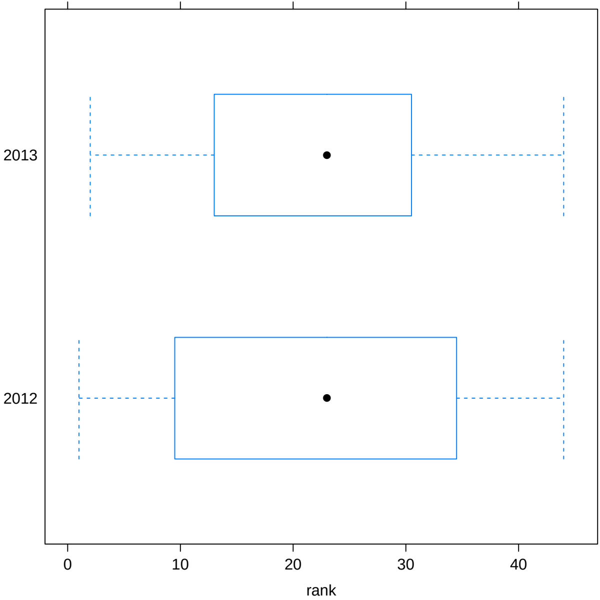
Summary: Local Search Algorithms
(as in [Hoos, Stützle, 2005])

For given problem instance π:

1. search space S_π
2. evaluation function $f_\pi : S \to \mathbf{R}$
3. neighborhood relation $\mathcal{N}_\pi \subseteq S_\pi \times S_\pi$
4. set of memory states M_π
5. initialization function $\texttt{init} : \emptyset \to S_\pi \times M_\pi)$
6. step function $\texttt{step} : S_\pi \times M_\pi \to S_\pi \times M_\pi$
7. termination predicate $\texttt{terminate} : S_\pi \times M_\pi \to \{\top, \bot\}$

Outline

1. GCP
    - CH for GCP
    - Code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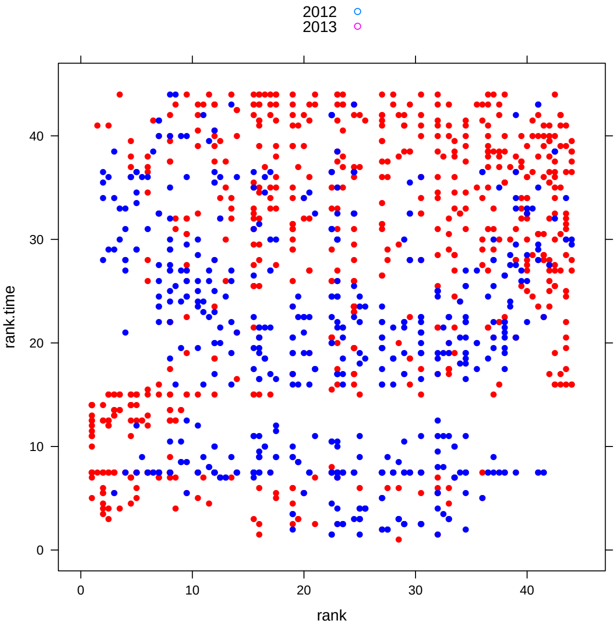
2012
0
10
20
30
40
rank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rank.time
rank
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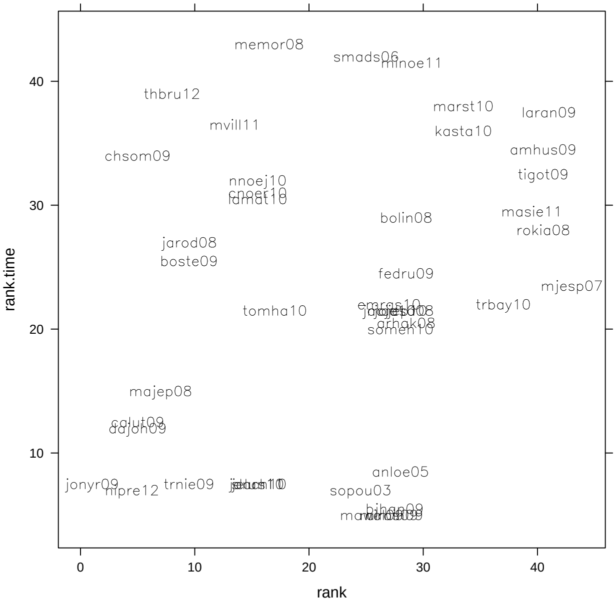
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cnoer10
lamat10
masie11
bolin08
rokia08
jarod08
boste09
fedru09
mjesp07
trbay10
tomha10
arhak08
someh10
majep08
calut09
dajon09
anloe05
jonyr09
nipre12
trnie09
sopou03
bihan09
rank.time
rank
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Construction Heuristics

- sequential heuristics
  1. choose a variable (vertex)
     a) static order: random (ROS),
        largest degree first, smallest degree last
     b) dynamic order: saturation degree (DSATUR) [Brélaz, 1979]
  2. choose a value (color): greedy heuristic

```
Procedure ROS
RandomPermutation π(Vertices);
forall i in 1, ..., n do
    v := π(i);
    select min{c : c not in saturated[v]};
    col[v] := c;
    add c in saturated[w] for all w adjacent v;
```

$\mathcal{O}(nk + m) \rightsquigarrow \mathcal{O}(n^2)$

```
Procedure DSATUR
select vertex v uncolored with max degree;
while uncolored vertices do
    select min{c : c not in saturated[v]};
    col[v] := c;
    add c in saturated[w] for all w adjacent v;
    select uncolored v with max size of
      saturated[v];
```

$\mathcal{O}(n(n + k) + m) \rightsquigarrow \mathcal{O}(n^2)$

- partitioning heuristics
  - recursive largest first (RLF) [Leighton, 1979]
    iteratively extract stable sets

Alternative form of pseudo-code

Procedure ROS
RandomPermutation π(Vertices);
forall i in $1, \ldots, n$ **do**
    v := π(i);
    selectMin {c : c not in saturated[v]} **do**
        col[v] := c;
        forall w in Vertices: adj[v,w] **do**
            saturated[w].insert(c);

Procedure DSATUR
RandomPermutation π(Vertices);
forall i in $1, \ldots, n$ **do**
    v := π(i);
    selectMin {c : c not in saturated[v]} **do**
        col[v] := c;
        forall w in Vertices: adj[v,w] **do**
            saturated[w].insert(c);

RLF [Leighton, 1979]

Procedure Recursive Largest First(G)
In $G = (V, E)$: input graph;
Out k : upper bound on $\chi(G)$;
Out c : a coloring $c : V \mapsto K$ of G;

$k \leftarrow 0$ **while** $|V| > 0$ **do**
  $k \leftarrow k + 1$ /* Use an additional color */
  FindStableSet(V, E, k) /* $G = (V, E)$ is reduced */
return k

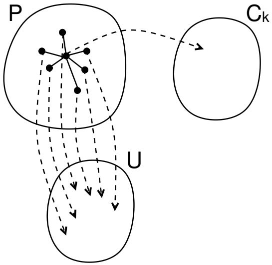
RLF

Key idea: extract stable sets trying to maximize edges removed.

Procedure FindStableSet(G, k)
In $G = (V, E)$: input graph
In k : color for current stable set
Var P : set of potential vertices for stable set
Var U : set of vertices that cannot go in current stable set

$P \leftarrow V$; $U \leftarrow \emptyset$;
forall $v \in P$ **do** $d_U(v) \leftarrow 0$; /* degree induced by U */
while P not empty **do**
    select v in P with max d_U;
    move v from P to C_k; $V \leftarrow V \setminus \{v\}$
    forall $w \in \delta_P(v)$ **do** /* neighbors of v in P */
        move w from P to U; $E \leftarrow E \setminus \{v, w\}$
        forall $u \in \delta_P(w)$ **do**
            $d_U(u) \leftarrow d_U(u) + 1$

$$\mathcal{O}(m + n\Delta^2) \rightsquigarrow \mathcal{O}(n^3)$$



Examples

```
import cotls;
include "loadDIMACS";
// int nv;
// int me;
// float alpha;
// bool adj[nv,nv];
range Vertices = 1..nv;
range Colors = 1..nv;
int nbc = Colors.getUp();

Solver<LS> m();

var{int} col[Vertices](m,Colors) := 1;
ConstraintSystem<LS> S(m);

forall (i in Vertices, j in Vertices: j>i && adj[i,j])
S.post(col[i] != col[j]);
S.close();

m.close();

// CONSTRUCTION HEURISTIC
set{int} dom[v in Vertices] = setof(c in Colors) true;
RandomPermutation perm(Vertices);
forall (i in 1..nv) {
   int v = perm.get();
   selectMin(c in dom[v])(c) {
      col[v] := c;
      forall(w in Vertices: adj[v,w])
         dom[w].delete(c);
   }
}
nbc = max(v in Vertices) col[v];
Colors = 1..nbc;
cout<<"Construction heuristic, done: "<<nbc<<" colors"<< endl;
```

code1.java/png code3.cpp

References

Brélaz D. (1979). **New methods to color the vertices of a graph**. *Communications of the ACM*, 22(4), pp. 251–256.

Leighton F.T. (1979). **A graph coloring algorithm for large scheduling problems**. *Journal of Research of the National Bureau of Standards*, 84(6), pp. 489–506.