DM841

Discrete Optimization

Part II

Lecture 2

Example
Gecode

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Outline

Example: Send More Money

Send + More = Money

You are asked to replace each letter by a different digit so that

	S	E	N	D	+
	M	O	R	E	=
M	O	N	E	Y	

is correct. Because S and M are the leading digits, they cannot be equal to the 0 digit.

Can you model this problem in IP/LS/CP?

Send More Money: CP model

SEND + MORE = MONEY

- $X_i \in \{0, \ldots, 9\}$ for all $i \in I = \{S, E, N, D, M, O, R, Y\}$

- Crypto constraint $\rightsquigarrow$ 1 equality constraint:

$$\begin{array}{rrrrrl} & 10^3 X_1 & +10^2 X_2 & +10 X_3 & +X_4 & + \\ & 10^3 X_5 & +10^2 X_6 & +10 X_7 & +X_2 & = \\ \hline 10^4 X_5 & +10^3 X_6 & +10^2 X_3 & +10 X_2 & +X_8 & \end{array}$$

- Each letter takes a different digit $\rightsquigarrow$ 1 inequality constraint

$$\texttt{alldifferent}([X_1, X_2, \ldots, X_8]).$$

(it substitutes 28 inequality constraints: $X_i \neq X_j, i, j \in I,\ i \neq j$)

ILP model + CP propagation

- $x_i \in \{0, \ldots, 9\}$ for all $i \in I = \{S, E, N, D, M, O, R, Y\}$
- $y_{ij} \in \{0, 1\}$ for all $i \in I$, $j \in J = \{0, \ldots, 9\}$

$$
\begin{array}{cccccc}
 & 10^3 x_1 & +10^2 x_2 & +10x_3 & +x_4 & + \\
 & 10^3 x_5 & +10^2 x_6 & +10x_7 & +x_2 & = \\
\hline
10^4 x_5 & +10^3 x_6 & +10^2 x_3 & +10x_2 & +x_8 &
\end{array}
$$

-

$$
\begin{aligned}
\sum_{j \in J} y_{ij} &= 1, & \forall i \in I, \\
\sum_{i \in I} y_{ij} &\leq 1, & \forall j \in J, \\
x_i &= \sum_{j \in J} j y_{ij}, & \forall i \in I.
\end{aligned}
$$

- Propagation adds valid inequalities:

$$LB(X_i) \leq x_i \leq UB(X_i) \text{ for all } i \in I$$

.

- H. Simonis' demo, slides 42-56

Send More Money: CP model

Gecode-python

```
from gecode import *

s = space()
letters = s.intvars(8,0,9)
S,E,N,D,M,O,R,Y = letters
s.rel(M,IRT_NQ,0)
s.rel(S,IRT_NQ,0)
s.distinct(letters)
C = [1000, 100, 10, 1,
     1000, 100, 10, 1,
     -10000, -1000, -100, -10, -1]
X = [S,E,N,D,
     M,O,R,E,
     M,O,N,E,Y]
s.linear(C,X, IRT_EQ, 0)
s.branch(letters, INT_VAR_SIZE_MIN, INT_VAL_MIN)
for s2 in s.search():
    print(s2.val(letters))
```

Send Most Money: CP model

Gecode-python

Optimization version:

$$\max \sum_{i \in I'} C_i X_i, \; I' = \{M, O, N, E, Y\}$$

```
from gecode import *

s = space()
letters = s.intvars(8,0,9)
S,E,N,D,M,O,T,Y = letters
s.rel(M,IRT_NQ,0)
s.rel(S,IRT_NQ,0)
s.distinct(letters)
C = [1000, 100, 10, 1,
     1000, 100, 10, 1,
     -10000, -1000, -100, -10, -1]
X = [S,E,N,D,
     M,O,S,T,
     M,O,N,E,Y]
s.linear(C,X,IRT_EQ,0)
money = s.intvar(0,99999)
s.linear([10000,1000,100,10,1],[M,O,N,E,Y], IRT_EQ, money)
s.maximize(money)
s.branch(letters, INT_VAR_SIZE_MIN, INT_VAL_MIN)
for s2 in s.search():
    print(s2.val(money), s2.val(letters))
```

Send More Money: CP model

MiniZinc

SEND-MORE-MONEY ≡ [DOWNLOAD]

```
include "alldifferent.mzn";

var 1..9: S;
var 0..9: E;
var 0..9: N;
var 0..9: D;
var 1..9: M;
var 0..9: O;
var 0..9: R;
var 0..9: Y;

constraint           1000 * S + 100 * E + 10 * N + D
                   + 1000 * M + 100 * O + 10 * R + E
       = 10000 * M + 1000 * O + 100 * N + 10 * E + Y;

constraint alldifferent([S,E,N,D,M,O,R,Y]);

solve satisfy;

output ["   ",show(S),show(E),show(N),show(D),"\n",
        "+  ",show(M),show(O),show(R),show(E),"\n",
        "= ",show(M),show(O),show(N),show(E),show(Y),"\n"];
```

References

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Smith B.M. (2006). **Modelling**. In *Handbook of Constraint Programming*, edited by F. Rossi, P. van Beek, and T. Walsh, chap. 11, pp. 377–406. Elsevier.

Williams H. and Yan H. (2001). **Representations of the all_different predicate of constraint satisfaction in integer programming**. *INFORMS Journal on Computing*, 13(2), pp. 96–103.