DM559

Linear and Integer Programming

Lecture 1
Introduction

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Outline

Outline

1. Course Organization

2. Preliminaries
    - Notation

Aims of the course

Learn about:

- both the theory and the practice of Linear Algebra
- one of the most important applications of Linear Algebra:
  - Mathematical optimization: linear programming
  - Discrete optimization: integer programming

$\rightsquigarrow$ You will apply the tools learned to solve real life problems using computer software

Contents of the Course (1/2)

(see Syllabus)

Linear Algebra: manipulation of matrices and vectors with some theoretical background

Linear Algebra

1. Matrices and vectors - Matrix algebra, Geometric insight
2. Systems of Linear Equations - Gaussian elimination
3. Matrix inversion and determinants
4. Rank, range and linear equations
5. Vector spaces
6. Linear Transformations - Matrix representation
7. Diagonalization - Eigenvalues and Eigenvectors

Contents of the Course (2/2)

(see Syllabus)

Linear Programming

1	Introduction - Linear Programming, Notation
2	Linear Programming, Simplex Method
3	Exception Handling
4	Duality Theory
5	Sensitivity
6	Revised Simplex Method

Integer Linear Programming

7	Modeling Examples, Good Formulations, Relaxations
8	Well Solved Problems
9	Network Optimization Models (Max Flow, Min cost flow, matching)
10	Cutting Planes & Branch and Bound
11	More on Modeling

Practical Information

Teacher: Marco Chiarandini (http://www.imada.sdu.dk/~marco/)
Instructor (Hold H1): Lasse Malm Lidegaard
Instructor (Hold H2): Marco Chiarandini

Schedule:

- Introductory classes: 44 hours (22 classes)
- Training classes: 50 hours
  - Exercises: 42 hours
  - Laboratory: 8 hours

Alternative views of the schedule:

- http://mitsdu.sdu.dk/skema, SDU Mobile
- Official course description (læserplaner)
- http://www.imada.sdu.dk/~marco/DM559

Communication Means

- BlackBoard (BB) $\Leftrightarrow$ Main Web Page (WWW)
  (link http://www.imada.sdu.dk/~marco/DM559)
- Announcements in BlackBoard
- Discussion Board in (BB) - allowed anonymous posting and rating
- Write to Marco (marco@imada.sdu.dk) and to instructor
- Ask peers
- You are welcome to visit me in my office in working hours (8-16)

$\rightsquigarrow$ It is good to ask questions!!

$\rightsquigarrow$ Let me know immediately when you think we should do things differently!

Books

Linear Algebra Part:

Le Steven J. Leon, Linear Algebra with Applications, 8th edition, Prentice Hall (2010).

Other books:

AH Martin Anthony and Michele Harvey, Linear Algebra, Concepts and Methods. 2012. Cambridge

AR Howard Anton and Chris Rorres. Elementary Linear Algebra. 11th Edition. 2015. Wiley

FSV Computing with Python: An introduction to Python for science and engineering Claus Führer, Jan Erik Solem, Olivier Verdier

Books

Linear and Integer Programming Part:

MG J. Matousek and B. Gartner. Understanding and Using Linear Programming. Springer Berlin Heidelberg, 2007

Wo L.A. Wolsey. Integer programming. John Wiley & Sons, New York, USA, 1998

Other books:

HL Frederick S Hillier and Gerald J Lieberman, Introduction to Operations Research, 9th edition, 2010

- ...

Course Material

Main Web Page (WWW) is the main reference for list of contents (ie[1], syllabus, pensum).

It Contains:

- slides
- list of topics and references
- exercises
- links
- software

[1] ie = id est, eg = exempli gratia, wrt = with respect to

Assessment

- 7.5 ECTS

- Three obligatory Assignments, pass/fail, evaluation by teacher
  practical exercises
  modeling + programming

- 4 hour written exam, 7-grade scale, external censor
  (theory part)
  similar to exercises in class and past exams
  in June

- (language: Danish and/or English)

Obligatory Assignments

- Small practical tasks must be passed to attend the written exam
- Best in groups of 2
- They require the use of Python + a MILP Solver (2nd part)
  Software available for all systems from the Main Web Page

Training Sessions

- Prepare them in advance to get out the most
- Best carried out in small groups
- Exam rehearsal (in June?)

Who is here?

60 registered in BlackBoard...

- Computer Science
  (2nd year, 4th semester)
- Applied Mathematics?
- Math-economy?

Prerequisites

- Calculus (MM501, MM502)

Previous evaluation

The majority of the students finds:

- indifferent the quality of the text books and exercises in class while satisfactory the lecture notes and the slides.
- satisfactory the preparation of the teacher while indifferent or dissatisfactory his pedagogical competencies. "M seems annoyed when asking something “trivial”"
- Marco as instructor has too high expectations and no understanding of where the students are.
- in general the course was pleasant and intellectually stimulating
- the volume of work necessary to complete this course means that it cannot all be thoroughly comprehended.
- there were too many exercises for the exercise sessions. The language in the exercises contains heavy mathematical notation and it is not easy to understand.
- motivation and goals were made clear
- the obligatory assignments were difficult
- the review process in Assignment 1 was positive
- low attendance was due to i) several assignments during the semester ii) the lagging behind due to lack of attendance in exercise sessions

Coding

- gives you the ability to create new and useful artifacts with just your mind and your fingers,
- allows you to have more control of your world as more and more of it becomes digital,
- is just fun.

It can also help you understand math!
Being able to turn procedural ideas into code and run the code on concrete examples gives you a great advantage in developing and reinforcing your understanding of mathematical concepts.
Beside:

- listening to lectures
- watching an instructor work through a derivation
- working through numerical examples by hand

you can learn by doing interacting with Python.

- Python 2.7 or 3.4?
- ipython (= interactive python)?

Outline

Overview

- Notation
- Martices and vectors:
    - matrix arithmetic operations (addition, subtraction, and multiplication)
    - scalar multiplication and transposition.

Outline

Sets

- A set is a collection of objects. eg.: $A = \{1, 2, 3\}$
- $A = \{n \mid n$ is a whole number and $1 \leq n \leq 3\}$
  ('$|$' reads 'such that')
- $B = \{x \mid x$ is a reader of this book$\}$
- $x \in A$
  x belongs to A
- set of no members: empty set, denoted $\emptyset$
- if a set S is a (proper) subset of a set T, we write $(S \subset T)$ $T \subseteq S$
  $\{1, 2, 5\} \subset \{1, 2, 4, 5, 6, 30\}$
- for two sets A and B, the union $A \cup B$ is $\{x \mid x \in A$ or $x \in B\}$
- for two sets A and B, the intersection $A \cap B$ is $\{x \mid x \in A$ and $x \in B\}$
  $\{1, 2, 3, 5\}$ and $B = \{2, 4, 5, 7\}$, then $A \cap B = \{2, 5\}$

Numbers

- set of real numbers: $\mathbb{R}$
- set of natural numbers: $\mathbb{N} = \{1, 2, 3, 4, ...\}$ (positive integers)
- set of all integers: $\mathbb{Z} = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$
- set of rational numbers: $\mathbb{Q} = \{p/q \mid p, q \in \mathbb{Z}, q \neq 0\}$
- set of complex numbers: $\mathbb{C}$
- absolute value (non-negative):

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a \leq 0 \end{cases}$$

  $|a + b| \leq |a| + |b|$, $a, b \in \mathbb{R}$

- the set $\mathbb{R}^2$ is the set of ordered pairs (x, y) of real numbers (eg, coordinates of a point wrt a pair of axes)

Basic Algebra

Elementary Algebra: the study of symbols and the rules for manipulating symbols It differs from arithmetic in the use of abstractions, such as using letters to stand for numbers that are either unknown or allowed to take on many values

- collecting up terms: eg. $2a + 3b - a + 5b = a + 8b$
- multiplication of variables: eg:

$$a(-b) - 3ab + (-2a)(-4b) = -ab - 3ab + 8ab = 4ab$$

- expansion of bracketed terms: eg:

$$\begin{aligned} -(a - 2b) &= -a + 2b, \\ (2x - 3y)(x + 4y) &= 2x^2 - 3xy + 8xy - 12y^2 \\ &= 2x^2 + 5xy - 12y^2 \end{aligned}$$

- $a^r a^s = a^{r+s}$, $(a^r)^s = a^{rs}$, $a^{-1} = 1/a^n$,
  $a^{1/n} = x \iff x^n = a$, $a^{m/n} = (a^{1/n})^m$

Quadratic Equations

- for a linear equation: $ax + b = 0$, $a, b \in \mathbb{R}$, a solution is a real number x for which the equation is true

- Quadratic equation

$$ax^2 + bx + c = 0, \qquad a \neq 0.$$

- Solved by factorization, eg:

$$x^2 - 6x + 5 = (x-1)(x-5) = 0$$

  then either $x - 1 = 0$ or $x - 5 = 0$.

- quadratic formula:

$$x_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \qquad x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

  the term $b^2 - 4ac$ is called discriminant

- Solutions from discriminant:
  - if $b^2 - 4ac > 0 \implies$ two real solutions
  - if $b^2 - 4ac = 0 \implies$ exactly one solution: $x = -b/(2a)$
  - if $b^2 - 4ac < 0 \implies$ no real solution but complex solutions

- Eg: i) $x^2 + 6x + 9 = 0$, and ii) $x^2 - 2x + 3 = 0$
  try using technique: completing the square

Polynomial Equations

- A polynomial of degree n in x is an expression of the form:

  $$P_n(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n,$$

  where the a_i are real constants, $a_n \neq 0$, and x is a real variable.

- $P_n(x) = 0$ has at most n solutions, eg:

  $$x^3 - 7x + 6 = (x-1)(x-2)(x+3) = 0,$$

  which are called roots or zeros of $P_n(x)$

- No general (closed) formula

- If α is a solution then $(x - \alpha)$ must be a factor for $P_n(x)$
  We find α by trial and error and then set $(x - \alpha)Q(x)$ where $Q(x)$ is a polynomial of degree $n - 1$

- Eg, $x^3 - 7x + 6$

Trigonometry

- sine and cosine functions, $\sin\theta$ and $\cos\theta$, geometrical meaning
- angles measured in radiants rather than degrees ($\pi = 180, \pi = 3.141...$)
- $\cos x = \sin(x + \pi/2)$
- $\sin^2\theta + \cos^2\theta = 1$
- $\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$
- $\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$