DM841

Discrete Optimization

Part I

Lecture 6

Examples
Consistency levels and global constraints

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Outline

Outline

Example: Sudoku

Model and solve the following Sudoku in MIP and CP

	4	3		8		2	5	
6								
					1		9	4
9					4		7	
			6		8			
	1		2					3
8	2		5					
								5
	3	4		9		7	1	

Sudoku: ILP model

Let y_{ijt} be equal to 1 if digit t appears in cell (i,j). Let N be the set $\{1,\ldots,9\}$, and let J_{kl} be the set of cells (i,j) in the 3×3 square in position k,l.

$$\sum_{j \in N} y_{ijt} = 1, \qquad \forall i, t \in N,$$

$$\sum_{j \in N} y_{jit} = 1, \qquad \forall i, t \in N,$$

$$\sum_{i,j \in J_{kl}} y_{ijt} = 1, \qquad \forall k, l = \{1,2,3\}, t \in N,$$

$$\sum_{t \in N} y_{ijt} = 1, \qquad \forall i, j \in N,$$

$$y_{i,j,a_{ij}} = 1, \qquad \forall i, j \in \text{ given instance.}$$

Sudoku: CP model

Model:

$$
\begin{aligned}
& X_{ij} \in N, && \forall i, j \in N, \\
& X_{ij} = a_{ij}, && \forall i, j \in \text{ given instance}, \\
& \text{alldifferent}([X_{1i}, \ldots, X_{9i}]), && \forall i \in N, \\
& \text{alldifferent}([X_{i1}, \ldots, X_{i9}]), && \forall i \in N, \\
& \text{alldifferent}(\{X_{ij} \mid ij \in J_{kl}\}), && \forall k, l \in \{1, 2, 3\}.
\end{aligned}
$$

Search: backtracking

Sudoku: CP model (revisited)

$$
\begin{aligned}
& X_{ij} \in N, && \forall i,j \in N,\\
& X_{ij} = a_t, && \forall i,j \in \text{ given instance},\\
& \text{alldifferent}([X_{1i},\ldots,X_{9i}]), && \forall i \in N,\\
& \text{alldifferent}([X_{i1},\ldots,X_{i9}]), && \forall i \in N,\\
& \text{alldifferent}(\{X_{ij} \mid ij \in J_{kl}\}), && \forall k,l \in \{1,2,3\}.
\end{aligned}
$$

Redundant Constraint:

$$
\begin{aligned}
& \sum_{j \in N} X_{ij} = 45, && \forall i \in N,\\
& \sum_{j \in N} X_{ji} = 45, && \forall i \in N,\\
& \sum_{ij \in J_{kl}} X_{ij} = 45, && k,l \in \{1,2,3\}.
\end{aligned}
$$

H. Simonis' demo, slides 10-56

Reminder: `alldifferent`

- Argument: list of variables
- Meaning: variables are pairwise different
- Reasoning: Forward Checking (FC)
  - When variable is assigned to value, remove the value from all other variables
  - If a variable has only one possible value, then it is assigned
  - If a variable has no possible values, then the constraint fails
  - Constraint is checked whenever one of its variables is assigned
  - Equivalent to decomposition into binary disequality constraints

Outline

Initial State (Forward Checking)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

Propagation Steps (Forward Checking)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9
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1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Forward Checking)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
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1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Forward Checking)

4	1 2 3 5 6 7 9	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2
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1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
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Propagation Steps (Forward Checking)

4	1 2 3 5 6 7 9	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
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1 3 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
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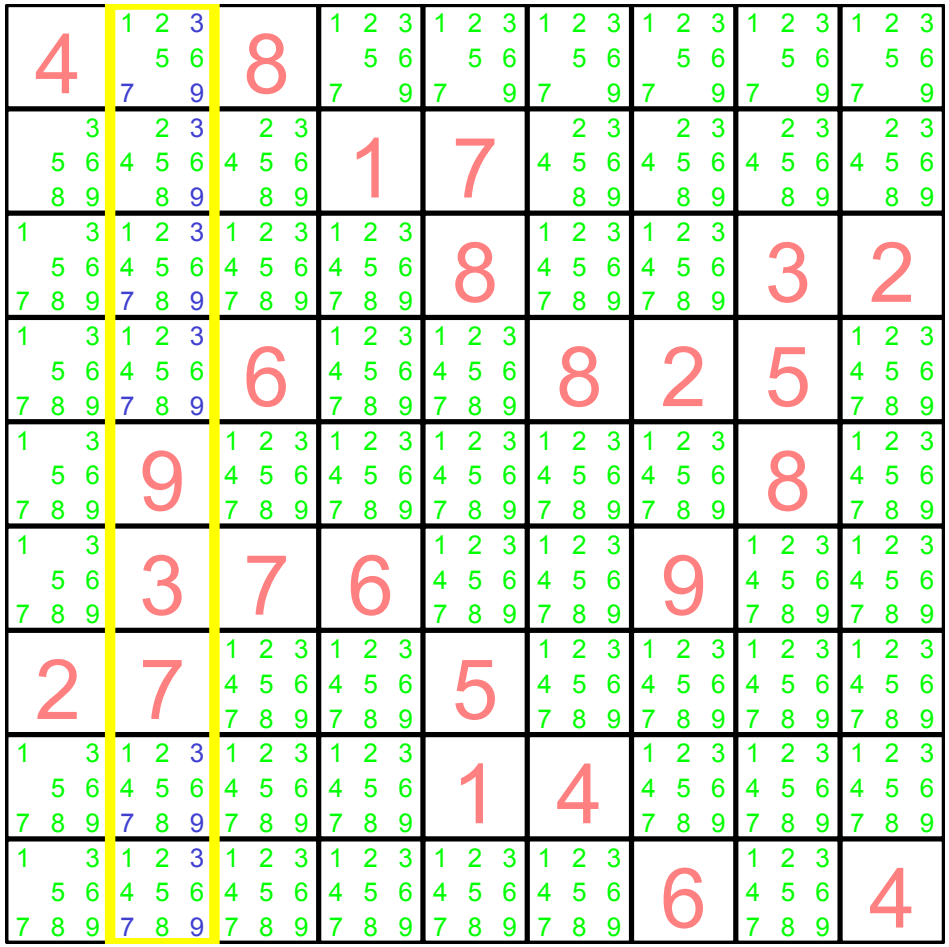
Propagation Steps (Forward Checking)



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Propagation Steps (Forward Checking)

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3 5 6 8 9	2 4 5 6 8	2 3 4 5 6 8 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
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1 3 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
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1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Forward Checking)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 6 8 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
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1 3 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
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1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Forward Checking)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
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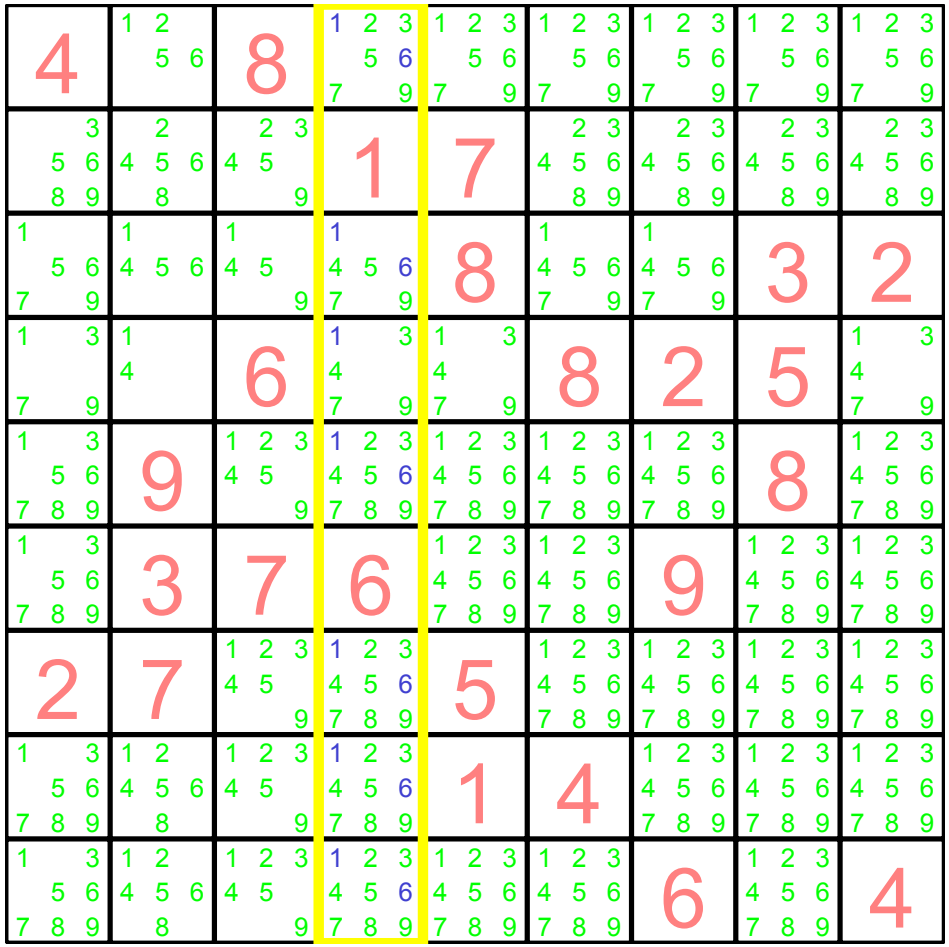
Propagation Steps (Forward Checking)



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Propagation Steps (Forward Checking)

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1 3 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
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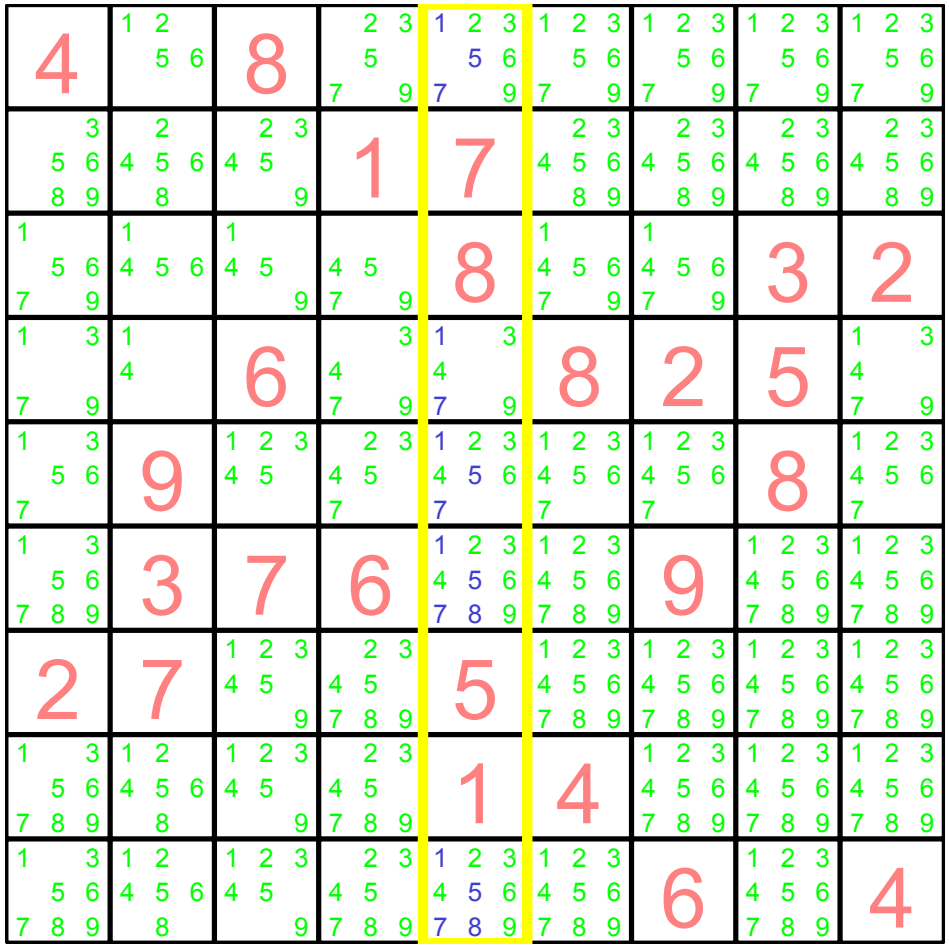
Propagation Steps (Forward Checking)



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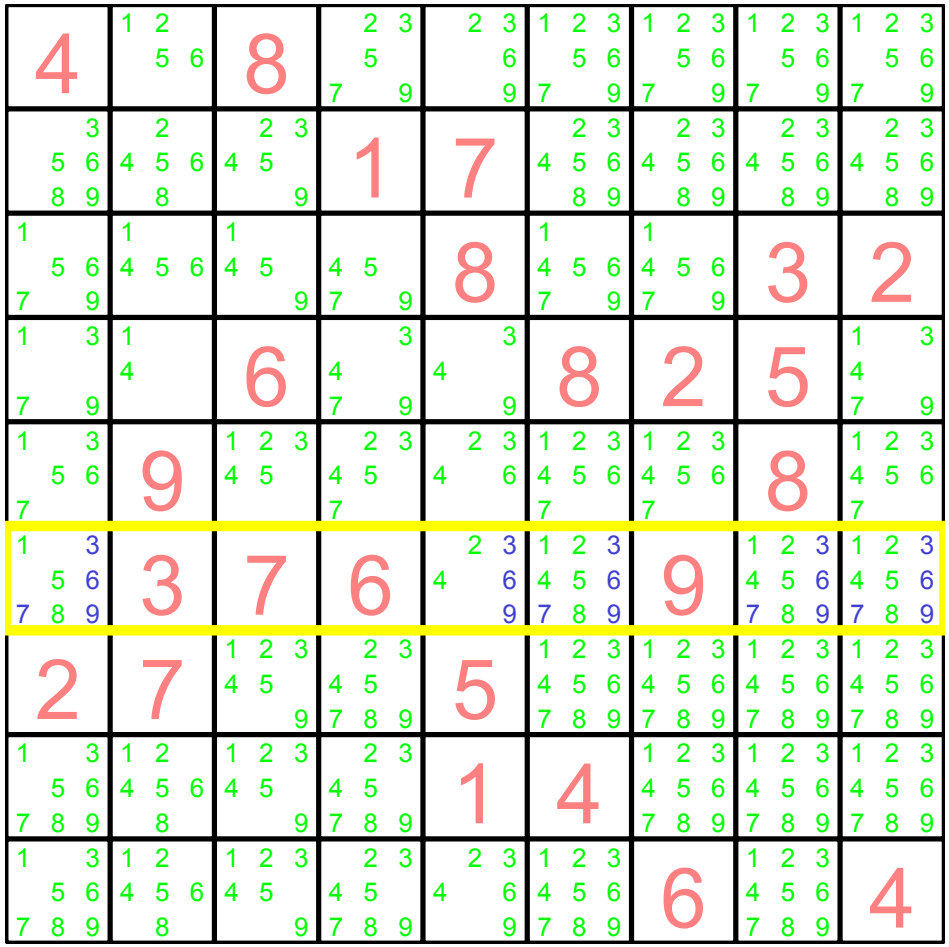
Propagation Steps (Forward Checking)



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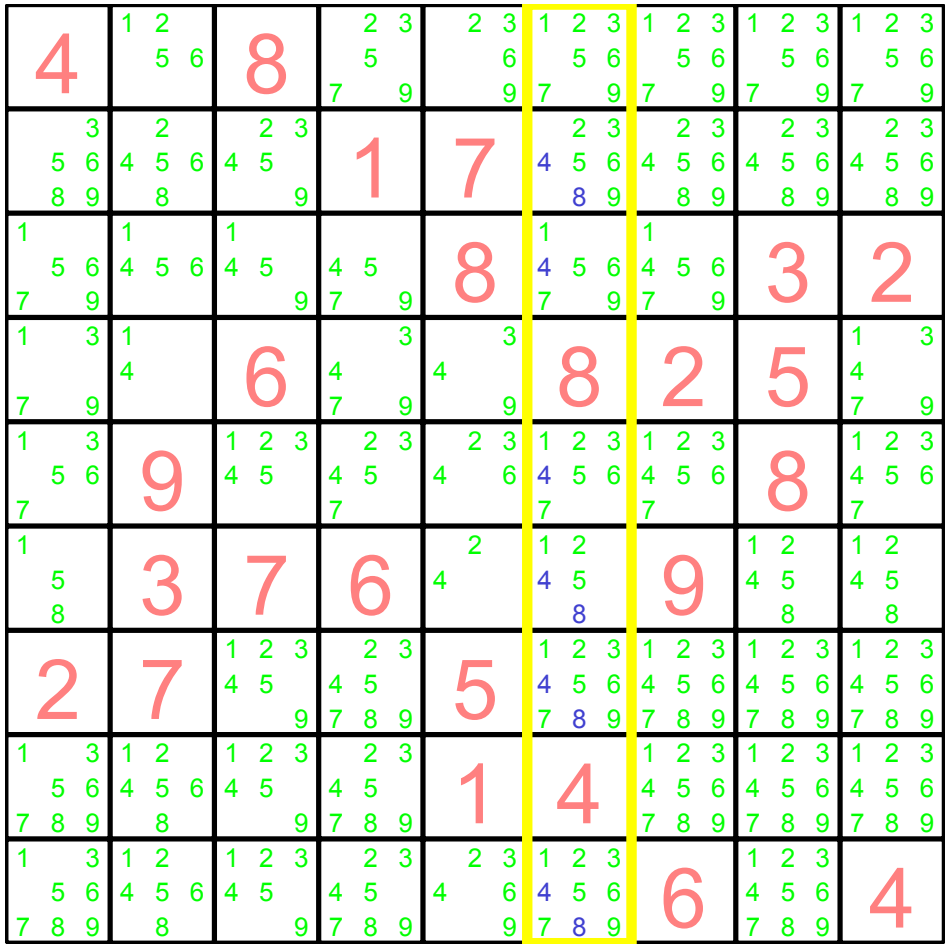
Propagation Steps (Forward Checking)



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Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 6 7 9	3	2
1 3 7 9	1 4	6	3 4 7 9	3 4 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 2 3 4 5 6 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4 5 8	1 2 4 5 8
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1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 7 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 6 7 9	3	2
1 3 7 9	1 4	6	3 4 7 9	3 4 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 2 3 4 5 6 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4 5 8	1 2 4 5 8
2	7	1 3 4 9	3 4 8 9	5	1 3 6 9	1 3 4 6 8 9	1 3 4 6 8 9	1 3 4 6 8 9
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1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 7 9	6	1 2 3 4 5 6 7 8 9	4

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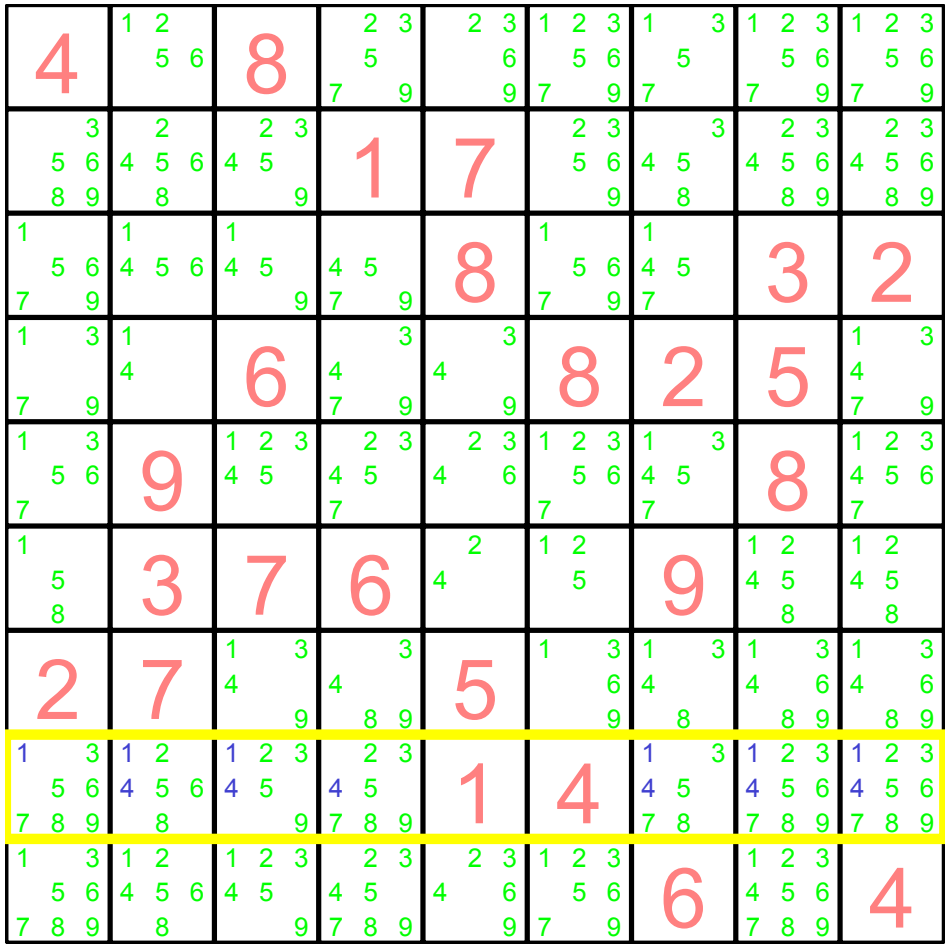
Propagation Steps (Forward Checking)



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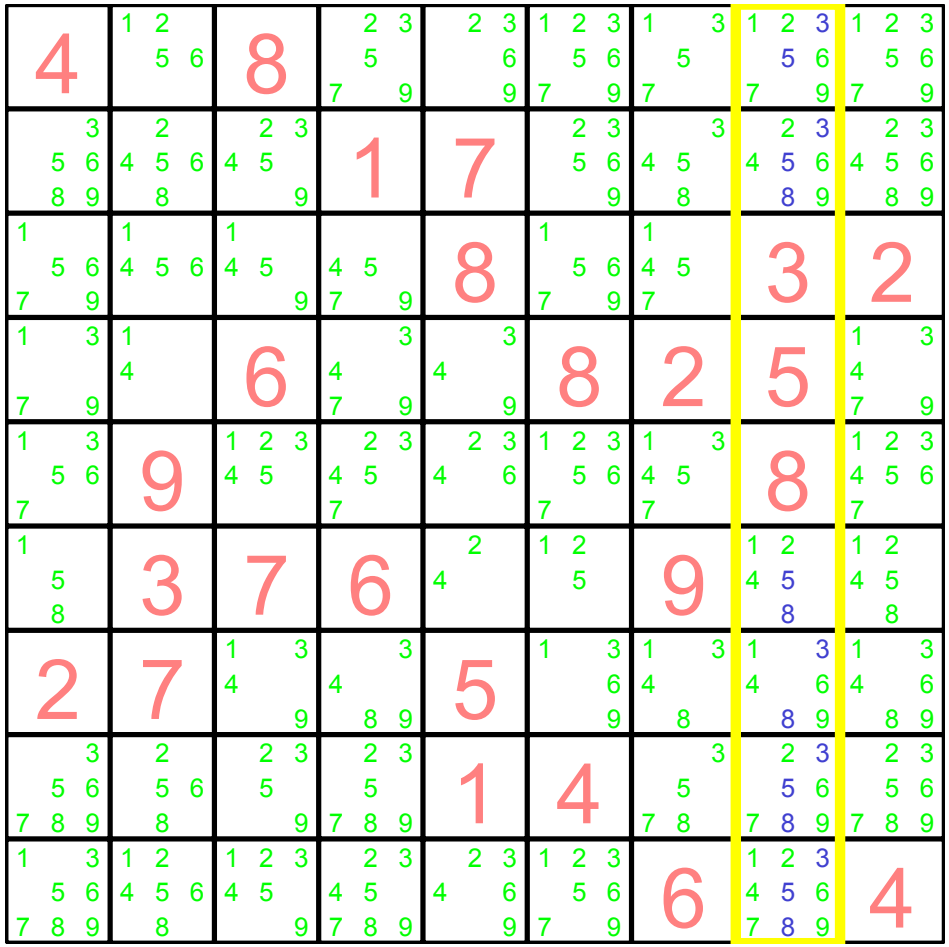
Propagation Steps (Forward Checking)



Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5 7	1 2 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	3 4 5 8	2 4 6 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2
1 3 7 9	1 4	6	3 4 7 9	3 4 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4	1 2 4 5 8
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1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 7 9	6	1 2 4 6 7 9	4

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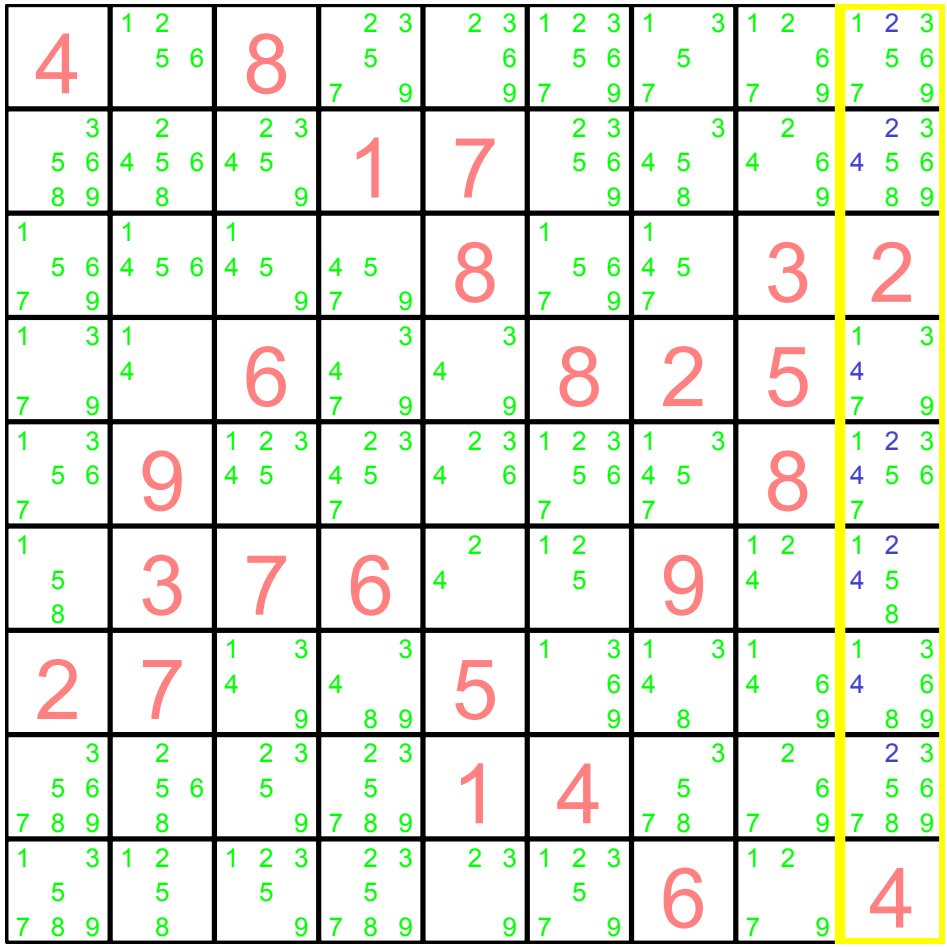
Propagation Steps (Forward Checking)



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Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5 7	1 2 6 7 9	1 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	3 4 5 8	2 4 6 9	3 5 6 8 9
1 5 6 7 9	1 4 5 6	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2
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1 3 5 7 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 9	1 2 3 5 7 9	6	1 2 7 9	4

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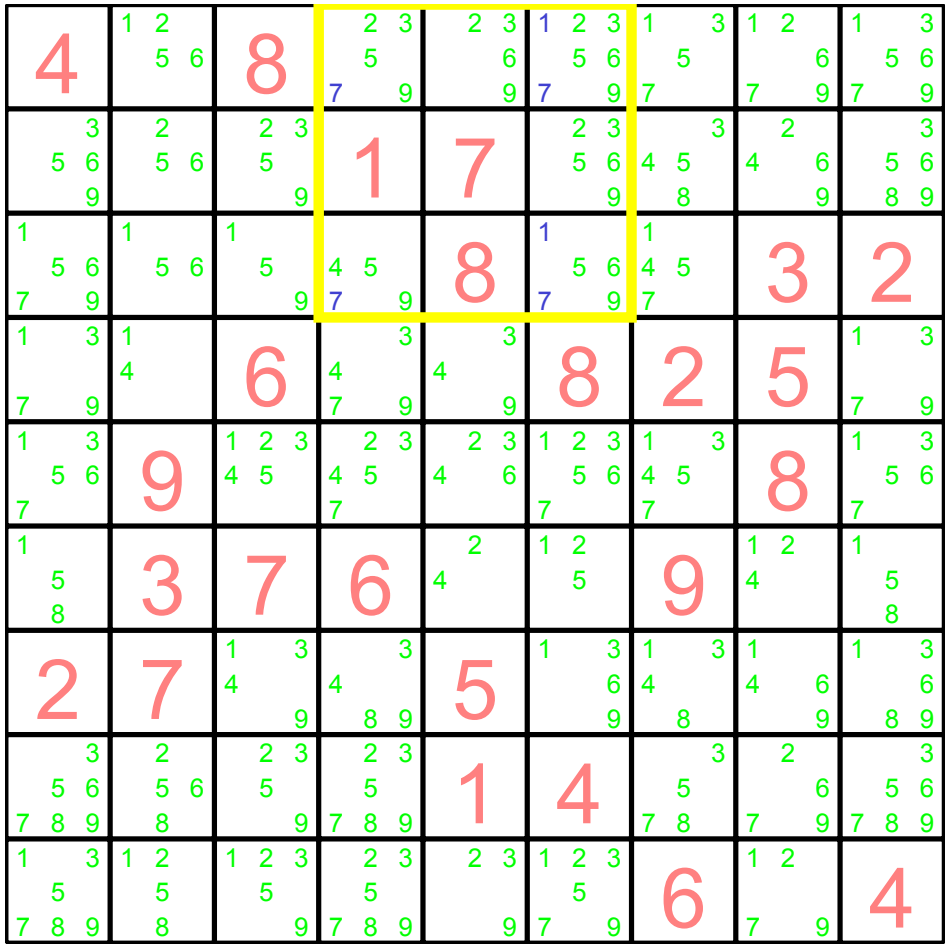
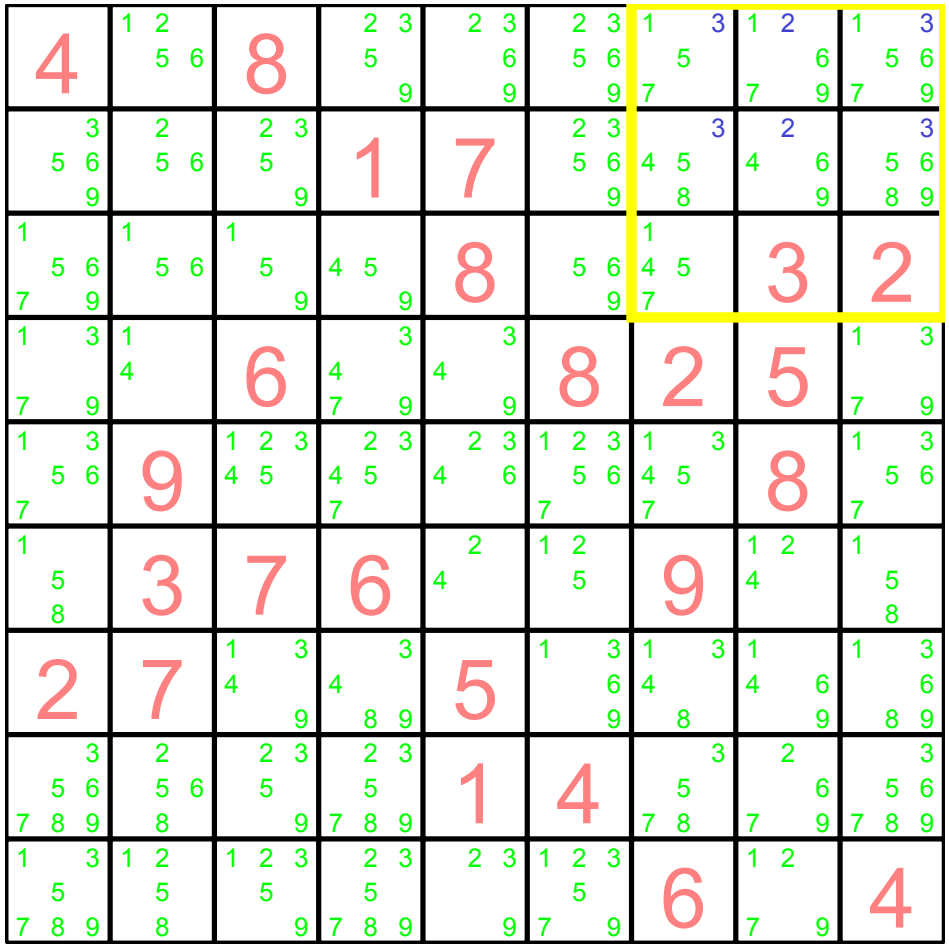
Propagation Steps (Forward Checking)



Propagation Steps (Forward Checking)



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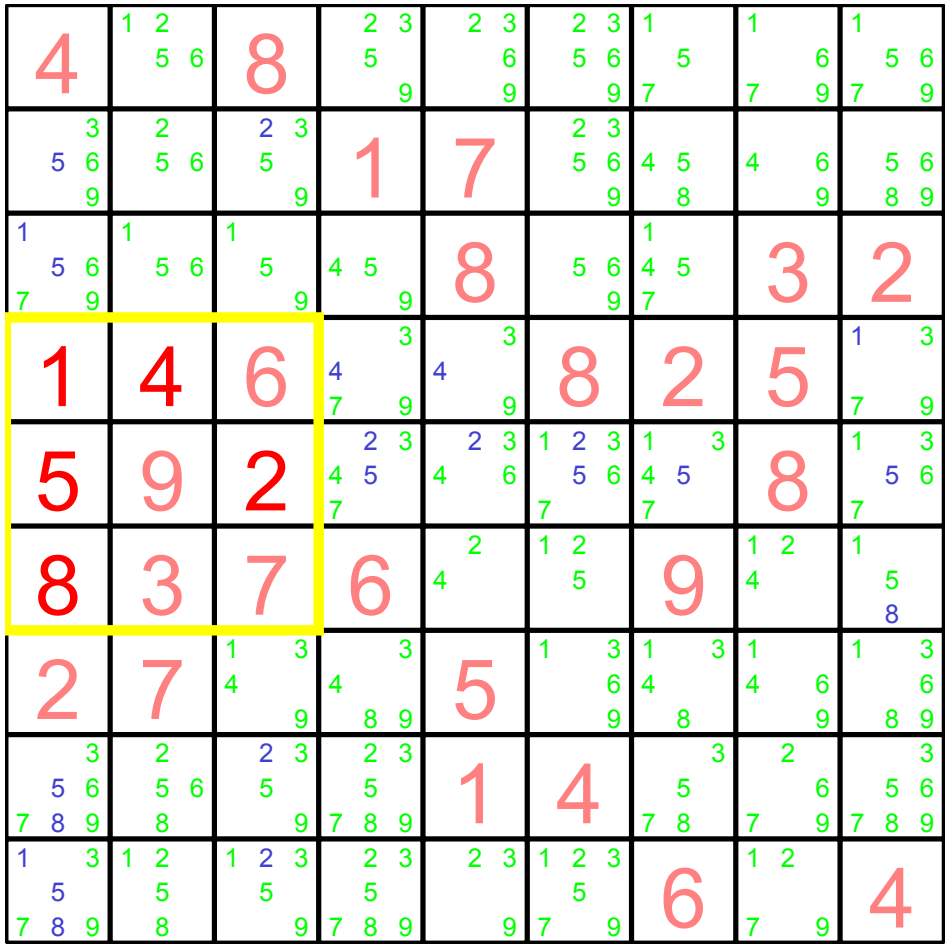
Propagation Steps (Forward Checking)



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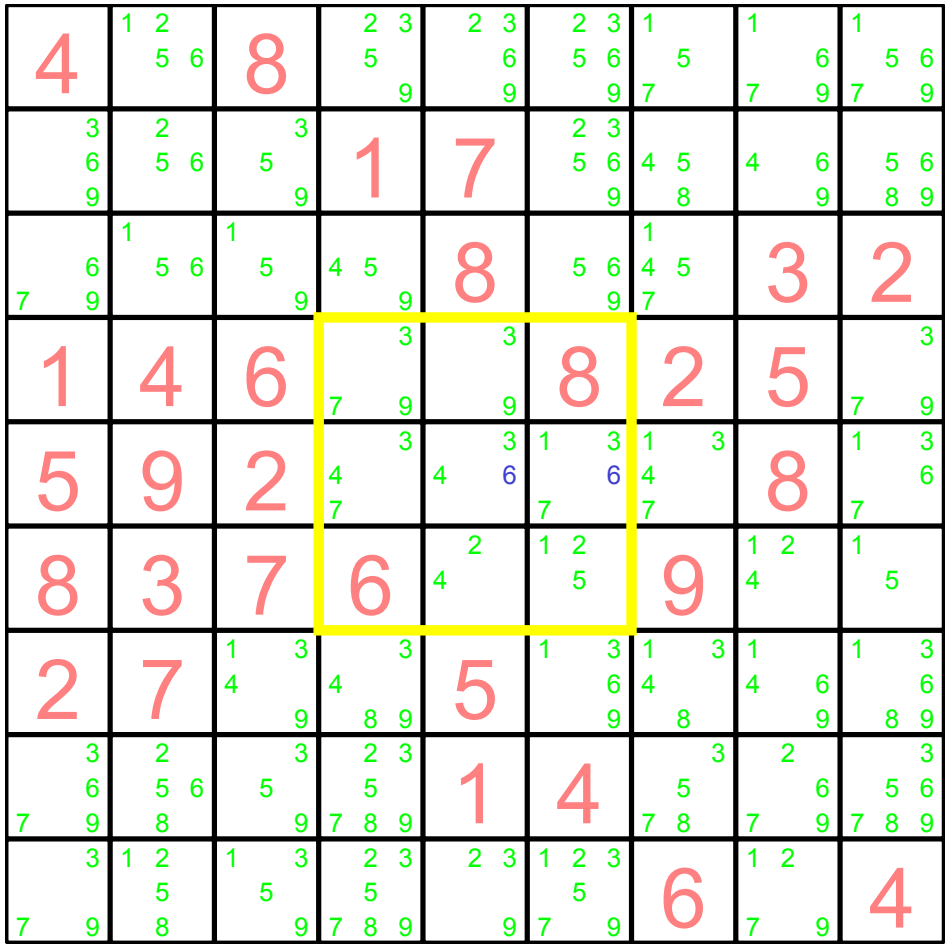
Propagation Steps (Forward Checking)



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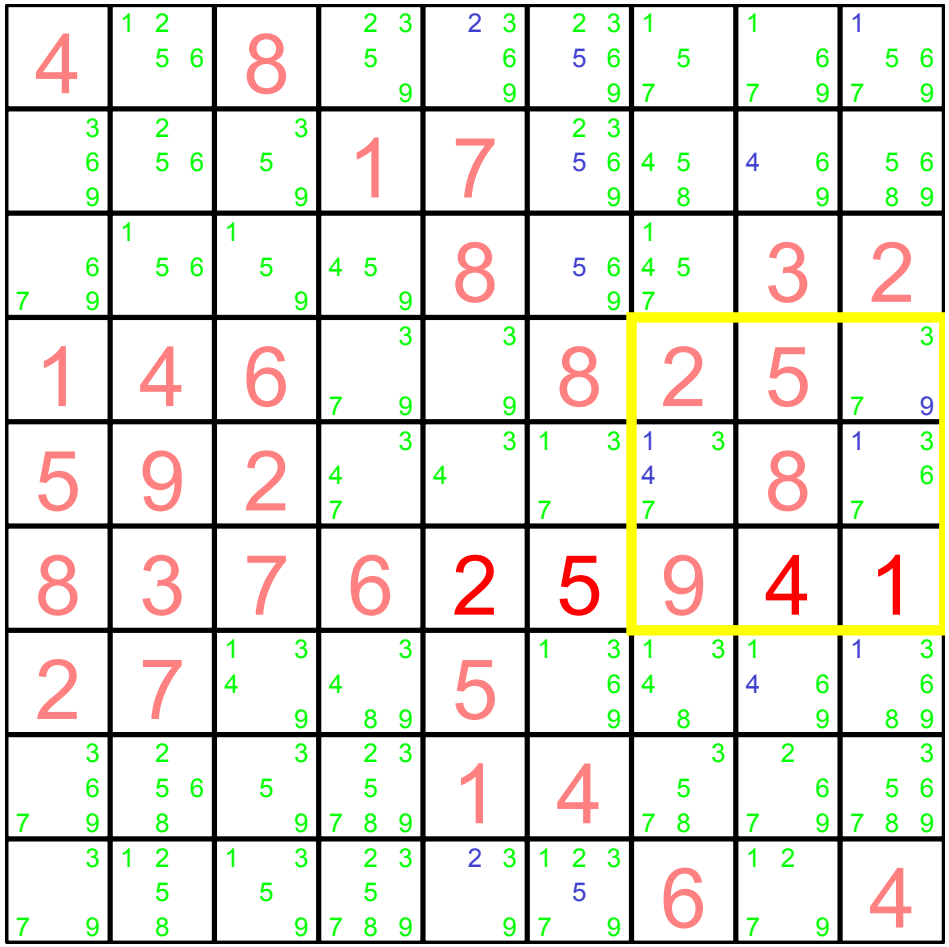

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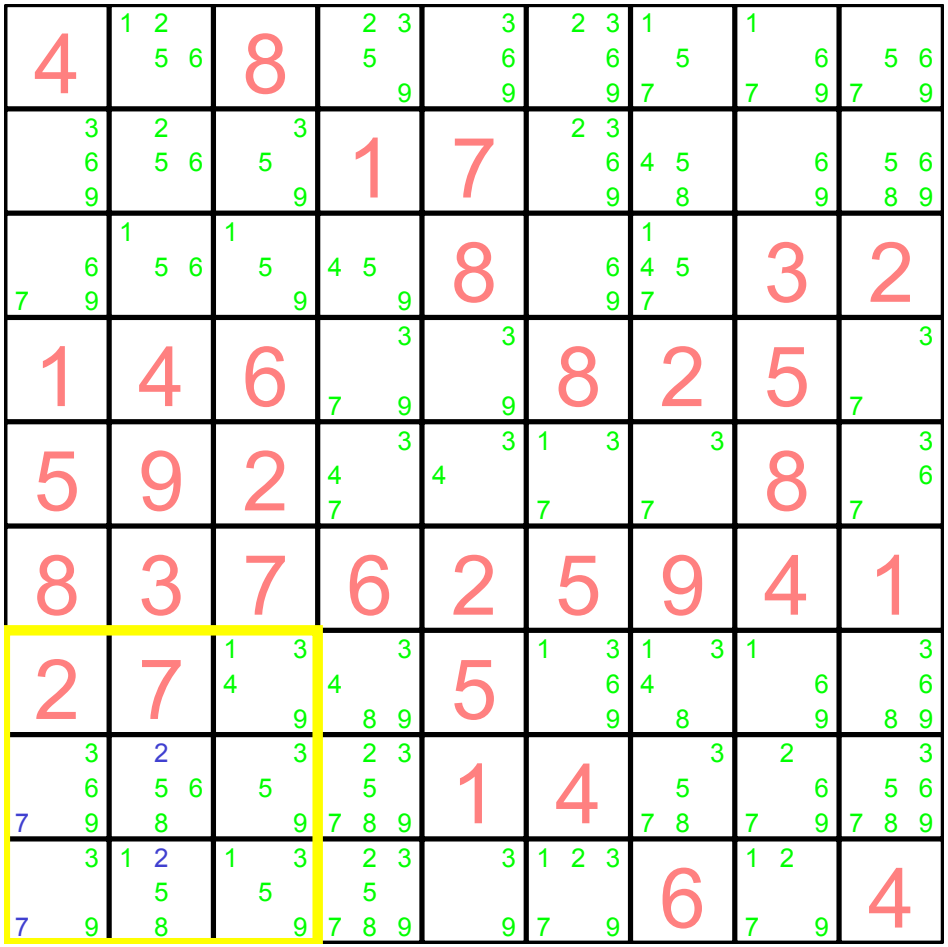
Propagation Steps (Forward Checking)



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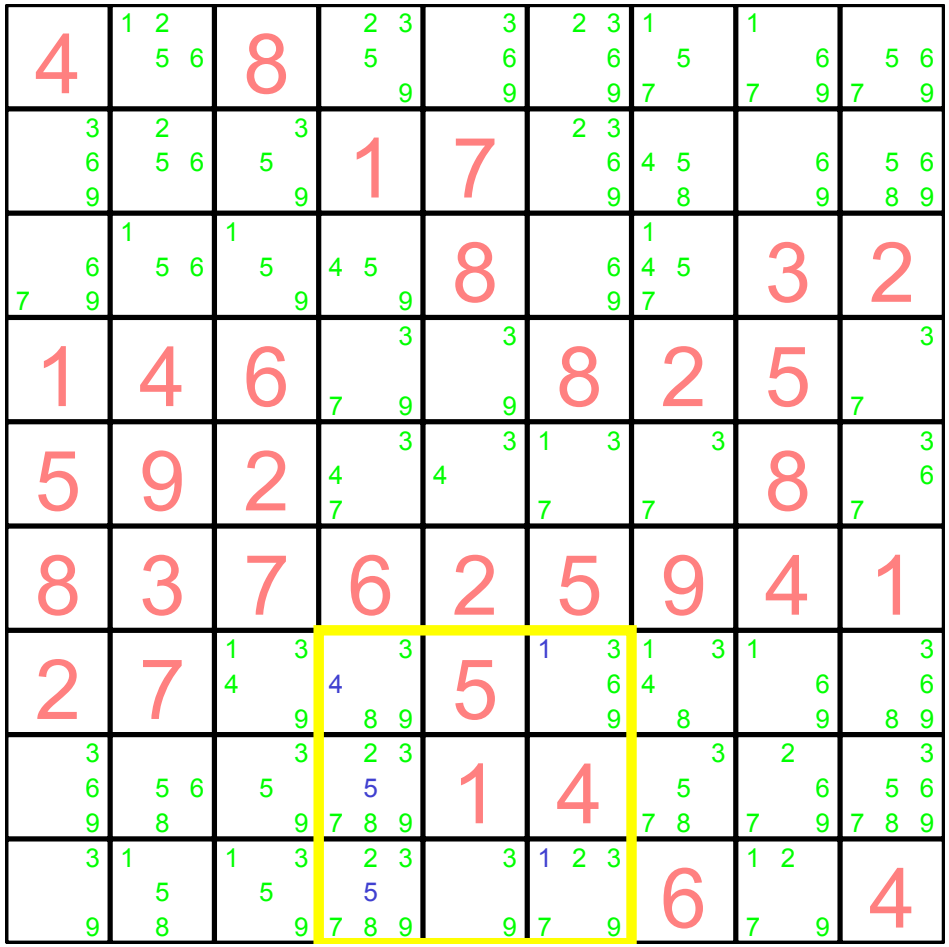
Propagation Steps (Forward Checking)



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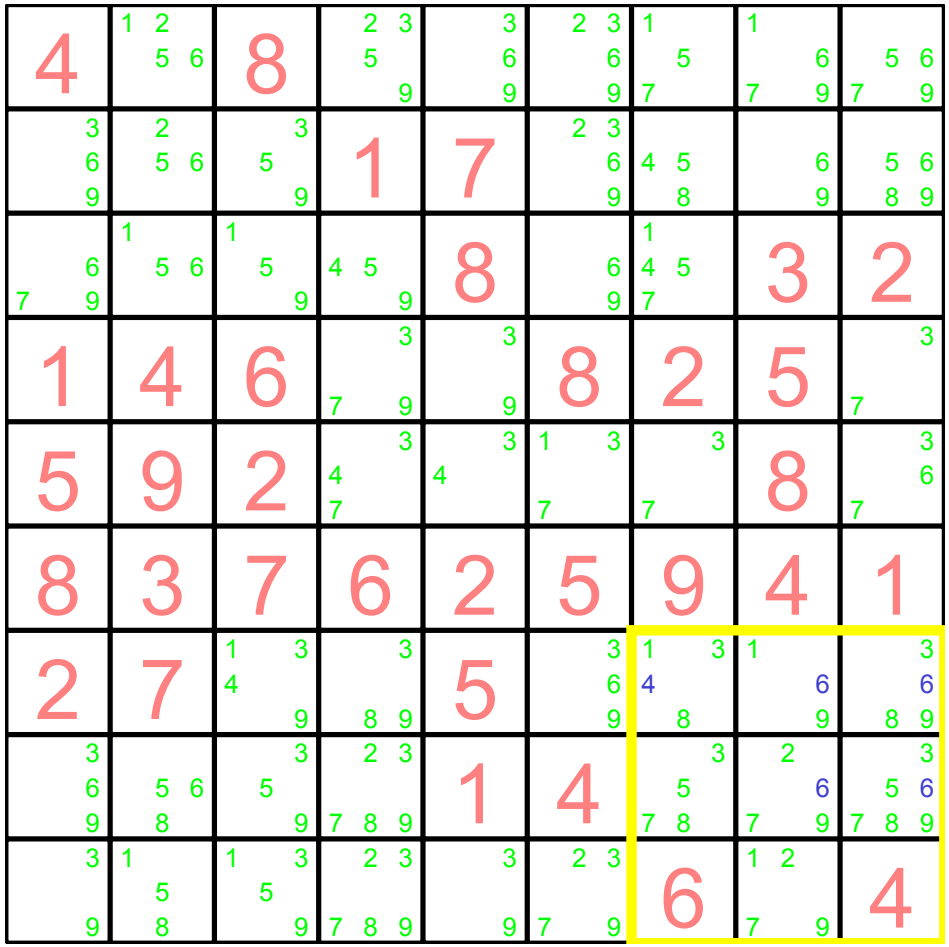
Propagation Steps (Forward Checking)



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Propagation Steps (Forward Checking)

4	1 2 5 6	8	2 3 5 9	3 6 9	2 3 6 9	1 5 7	1 6 7 9	5 6 7 9
3 6 9	2 5 6	3 5 9	1	7	2 3 6 9	4 5 8	6 9	5 6 8 9
6 7 9	1 5 6	1 5 9	4 5 9	8	6 9	1 4 5 7	3	2
1	4	6	3 7 9	3 9	8	2	5	3 7
5	9	2	3 4 7	3 4	1 3 7	3 7	8	3 6 7
8	3	7	6	2	5	9	4	1
2	7	1 3 4 9	3 8 9	5	3 6 9	1 3 8	1 9	3 8 9
3 6 9	5 6 8	3 5 9	2 3 7 8 9	1	4	3 5 7 8	2 7 9	3 5 7 8 9
3 9	1 5 8	1 3 5 9	2 3 7 8 9	3 9	2 3 7 9	6	1 2 7 9	4

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After Setup (Forward Checking)

4	1 2 5 6	8	2 3 5 9	3 6 9	2 3 6 9	1 5 7	1 6 7 9	5 6 7 9
3 6 9	2 5 6	3 5 9	1	7	2 3 6 9	4 5 8	6 9	5 6 8 9
6 7 9	1 5 6	1 5 9	4 5 9	8	6 9	1 4 5 7	3	2
1	4	6	3 7 9	3 9	8	2	5	3 7
5	9	2	3 4 7	3 4	1 3 7	3 7	8	3 6 7
8	3	7	6	2	5	9	4	1
2	7	1 3 4 9	3 8 9	5	3 6 9	1 3 8	1 9	3 8 9
3 6 9	5 6 8	3 5 9	2 3 7 8 9	1	4	3 5 7 8	2 7 9	3 5 7 8 9
3 9	1 5 8	1 3 5 9	2 3 7 8 9	3 9	2 3 7 9	6	1 2 7 9	4

Outline

Can we do better?

- The alldifferent constraint is missing propagation
  - How can we do more propagation?
  - Do we know when we derive all possible information from the constraint?
- Constraints only interact by changing domains of variables

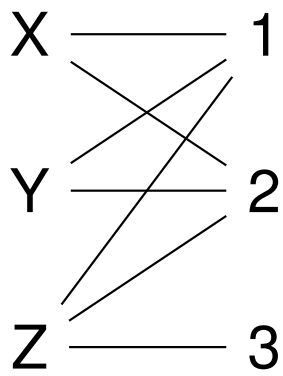
A Simpler Example

```
:-lib(ic).

top:-
    X :: 1..2,
    Y :: 1..2,
    Z :: 1..3,
    alldifferent([X,Y,Z]),
    writeln([X,Y,Z]).
```

Using Forward Checking

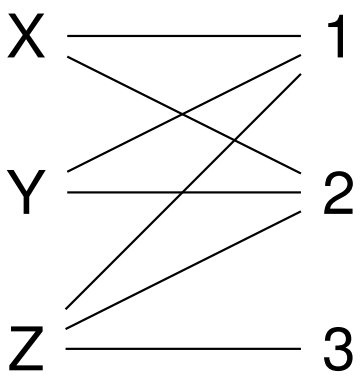
- No variable is assigned
- No reduction of domains
- But, values 1 and 2 can be removed from Z
- This means that Z is assigned to 3

Visualization of alldifferent as Graph



- Show problem as graph with two types of nodes
  - Variables on the left
  - Values on the right
- If value is in domain of variable, show link between them
- This is called a *bipartite* graph

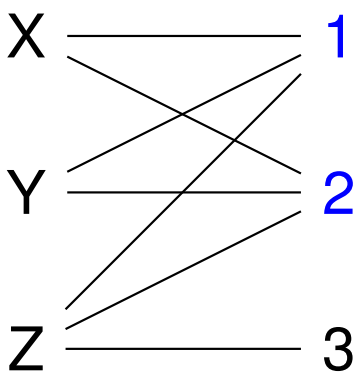
A Simpler Example



Value Graph for

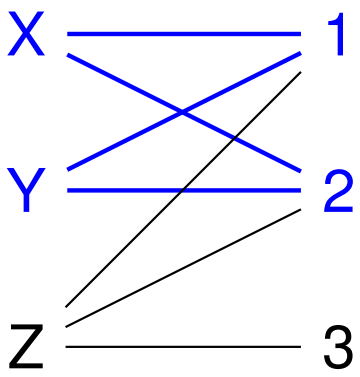
```
X :: 1..2,

Y :: 1..2,

Z :: 1..3
```

A Simpler Example



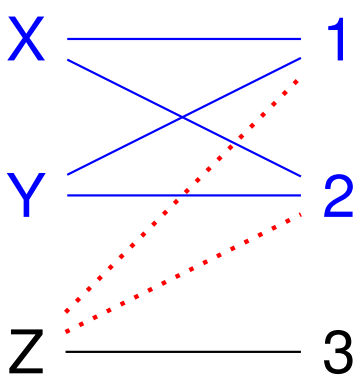
Check interval [1,2]

A Simpler Example



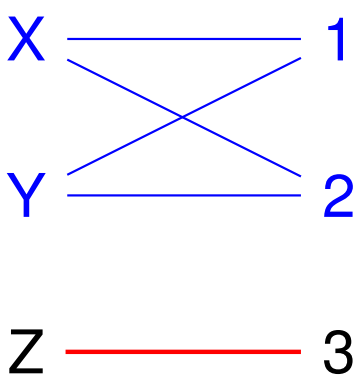
- Find variables completely contained in interval
- There are two: X and Y
- This uses up the capacity of the interval

A Simpler Example



No other variable can use that interval

A Simpler Example



Only one value left in domain of Z, this can be assigned

Idea (Hall Intervals)

- Take each interval of possible values, say size N
- Find all K variables whose domain is completely contained in interval
- If $K > N$ then the constraint is infeasible
- If $K = N$ then no other variable can use that interval
- Remove values from such variables if their bounds change
- If $K < N$ do nothing
- Re-check whenever domain bounds change

Implementation

- Problem: Too many intervals ($O(n^2)$) to consider
- Solution:
  - Check only those intervals which update bounds
  - Enumerate intervals incrementally
  - Starting from lowest(highest) value
  - Using sorted list of variables
- Complexity: $O(n \log(n))$ in standard implementations
- Important: Only looks at min/max bounds of variables

Bounds Consistency

Definition

A constraint achieves *bounds consistency*, if for the lower and upper bound of every variable, it is possible to find values for all other variables between their lower and upper bounds which satisfy the constraint.

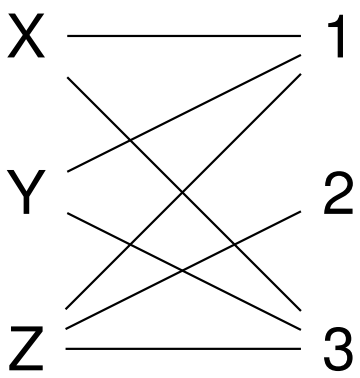
Can we do better?

- Bounds consistency only considers min/max bounds
- Ignores “holes” in domain
- Sometimes we can improve propagation looking at those holes

Another Simple Example

```
:-lib(ic).

top:-
    X :: [1,3],
    Y :: [1,3],
    Z :: 1..3,
    alldifferent([X,Y,Z]),
    writeln([X,Y,Z]).
```

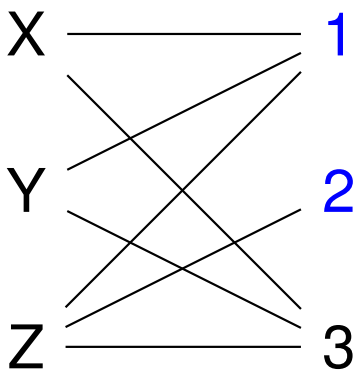
Another Simple Example



Value Graph for

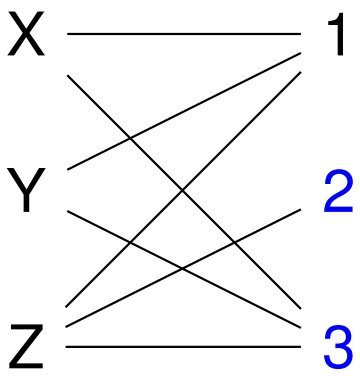
```
X :: [1,3],

Y :: [1,3],

Z :: 1..3
```

Another Simple Example



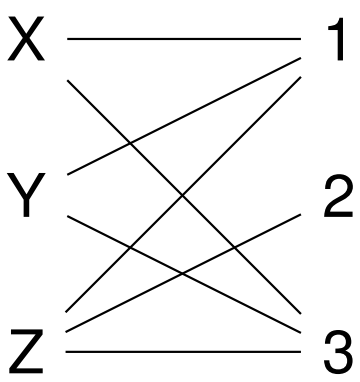
- Check interval [1,2]
- No domain of a variable completely contained in interval
- No propagation

Another Simple Example



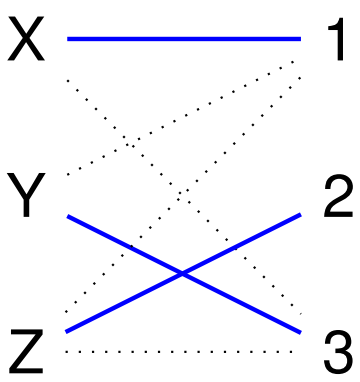
- Check interval [2,3]
- No domain of a variable completely contained in interval
- No propagation

Another Simple Example



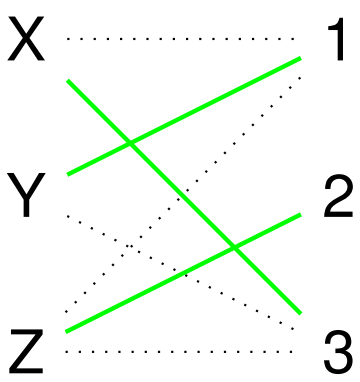
But, more propagation is possible, there are only two solutions

Another Simple Example



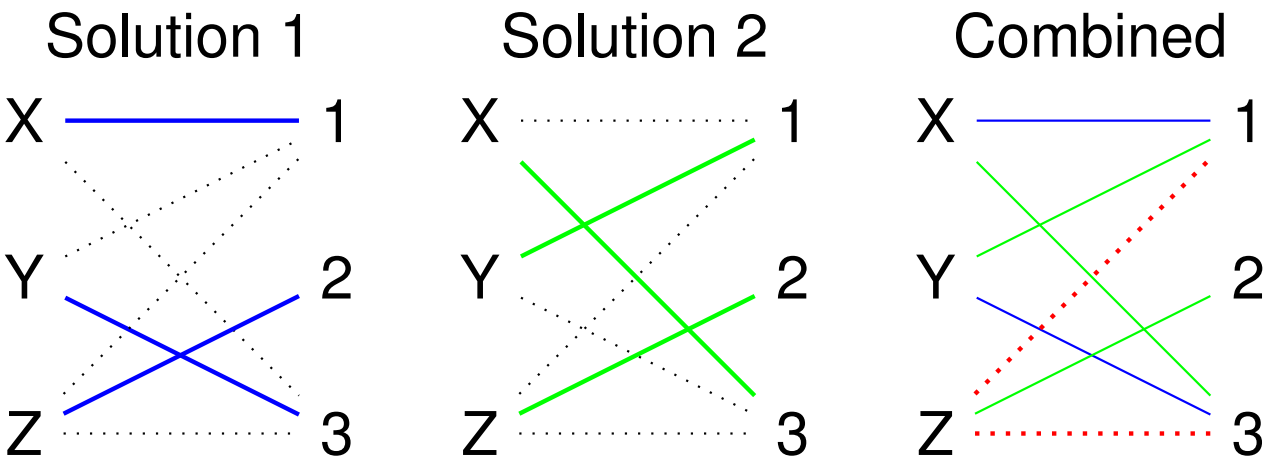
Solution 1: assignment in blue

Another Simple Example



Solution 2: assignment in green

Another Simple Example



Combining solutions shows that Z=1 and Z=3 are not possible.

Another Simple Example

Can we deduce this without enumerating solutions?

Solutions and maximal matchings

- A *Matching* is subset of edges which do not coincide in any node
- No matching can have more edges than number of variables
- Every solution corresponds to a *maximal matching* and vice versa
- If a link does not belong to some maximal matching, then it can be removed

Implementation

- Possible to compute all links which belong to some matching
  - Without enumerating all of them!
- Enough to compute **one** maximal matching
- Requires algorithm for *strongly connected components*
- Extra work required if more values than variables
- All links (values in domains) which are not supported can be removed
- Complexity: $O(n^{1.5}d)$

Domain Consistency

Definition

A constraint achieves *domain consistency*, if for every variable and for every value in its domain, it is possible to find values in the domains of all other variables which satisfy the constraint.

- Also called *generalized arc consistency (GAC)*
- or *hyper arc consistency*

Can we still do better?

- NO! This extracts all information from this one constraint
- We could perhaps improve speed, but not propagation
- But possible to use different model
- Or model interaction of multiple constraints

Should all constraints achieve domain consistency?

- Domain consistency is usually more expensive than bounds consistency
  - Overkill for simple problems
  - Nice to have choices
- For some constraints achieving domain consistency is NP-hard
  - We have to live with more restricted propagation

Initial State (Bounds Consistency)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

Propagation Steps (Bounds Consistency)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
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2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

Skip Animation

Propagation Steps (Bounds Consistency)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
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1 2 3 4 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Bounds Consistency)

4	1 2 3 5 6 7 9	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
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Propagation Steps (Bounds Consistency)

4	1 2 3 5 6 7 9	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
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Propagation Steps (Bounds Consistency)

4	1 2 3 5 6 7 9	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
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Propagation Steps (Bounds Consistency)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
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1 3 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
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Propagation Steps (Bounds Consistency)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
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Propagation Steps (Bounds Consistency)

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Propagation Steps (Bounds Consistency)

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Propagation Steps (Bounds Consistency)

4	1 2 5 6	8	2 3 5 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
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1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Bounds Consistency)

4	1 2 5 6	8	2 3 5 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
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1 3 7 9	1 4	6	3 4 7 9	1 3 4 7 9	8	2	5	1 3 4 7 9
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1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Bounds Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
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1 3 7 9	1 4	6	3 4 7 9	3 4 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 4 5 6 7	1 2 3 4 5 6 7	8	1 2 3 4 5 6 7
1 3 5 6 7 8 9	3	7	6	2 3 4 6 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
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1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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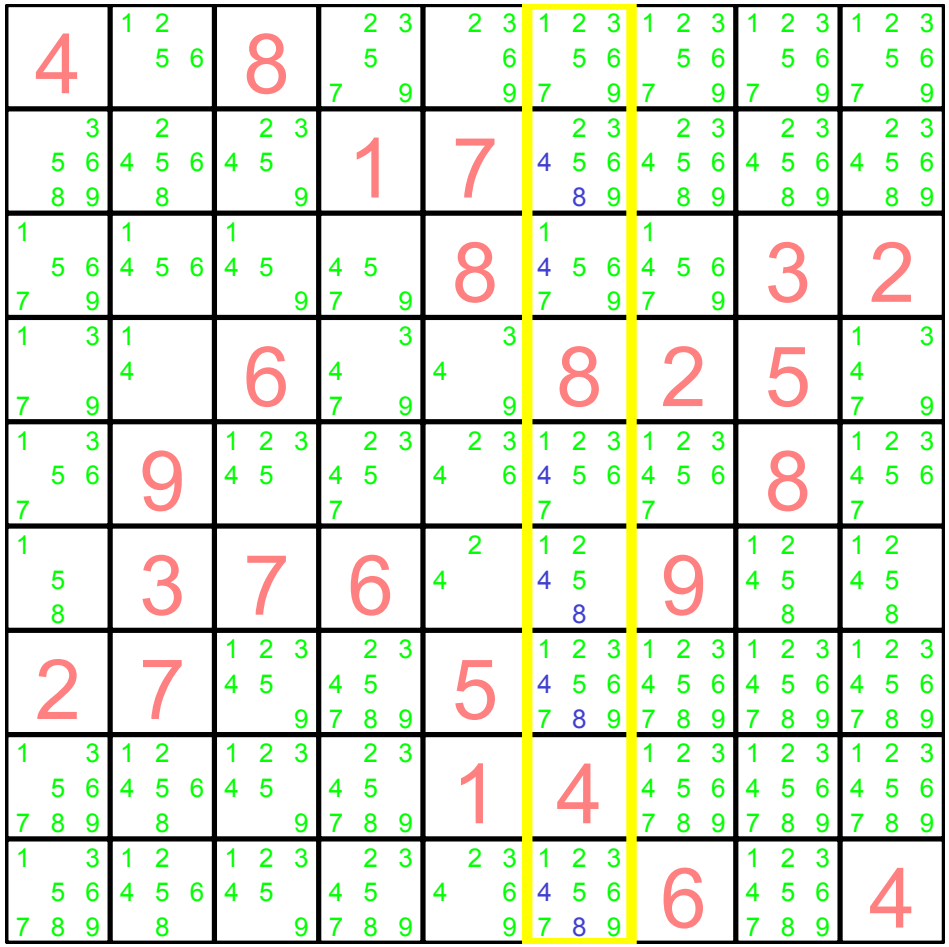
Propagation Steps (Bounds Consistency)



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Propagation Steps (Bounds Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
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1 3 7 9	1 4	6	3 4 7 9	3 4 9	8	2	5	1 3 4 7 9
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1 5 8	3	7	6	2 4	1 2 5	9	1 2 4 5 8	1 2 4 5 8
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Propagation Steps (Bounds Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
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1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 7 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Bounds Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5 7	1 2 3 5 6 7 9	1 2 3 5 6 7 9
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1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 7 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Bounds Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5 7	1 2 3 5 6 7 9	1 2 3 5 6 7 9
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Propagation Steps (Bounds Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5 7	1 2 6 7 9	1 2 3 5 6 7 9
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Propagation Steps (Bounds Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5 7	1 2 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	3 4 5 8	2 4 6 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2
1 3 7 9	1 4	6	3 4 7 9	3 4 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4	1 2 4 5 8
2	7	1 3 4 9	3 4 8 9	5	1 3 6 9	1 3 4 8	1 4 6 9	1 3 4 6 8 9
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	3 5 7 8	2 6 7 9	2 3 5 6 7 8 9
1 3 5 7 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 9	1 2 3 5 7 9	6	1 2 7 9	4

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Propagation Steps (Bounds Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5 7	1 2 6 7 9	1 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	3 4 5 8	2 4 6 9	3 5 6 8 9
7	1 4 5 6	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2
1 3 7 9	1 4	6	3 4 7 9	3 4 9	8	2	5	1 3 7 9
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 3 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4	1 5 8
2	7	1 3 4 9	3 4 8 9	5	1 3 6 9	1 3 4 8	1 4 6 9	1 3 6 8 9
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	3 5 7 8	2 6 7 9	3 5 6 7 8 9
1 3 5 7 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 9	1 2 3 5 7 9	6	1 2 7 9	4

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Propagation Steps (Bounds Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5 7	1 2 6 7 9	1 3 5 6 7 9
3 5 6 9	2 5 6	2 3 5 9	1	7	2 3 5 6 9	3 4 5 8	2 4 6 9	3 5 6 8 9
7	1 5 6	1 5 9	4 5 9	8	1 5 6 9	1 4 5	3	2
1 3 9	1 4	6	3 4 7 9	3 4 9	8	2	5	1 3 7 9
1 3 5 6	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 3 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4	1 5 8
2	7	1 3 4 9	3 4 8 9	5	1 3 6 9	1 3 4 8	1 4 6 9	1 3 6 8 9
3 5 6 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	3 5 7 8	2 6 7 9	3 5 6 7 8 9
1 3 5 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 9	1 2 3 5 7 9	6	1 2 7 9	4

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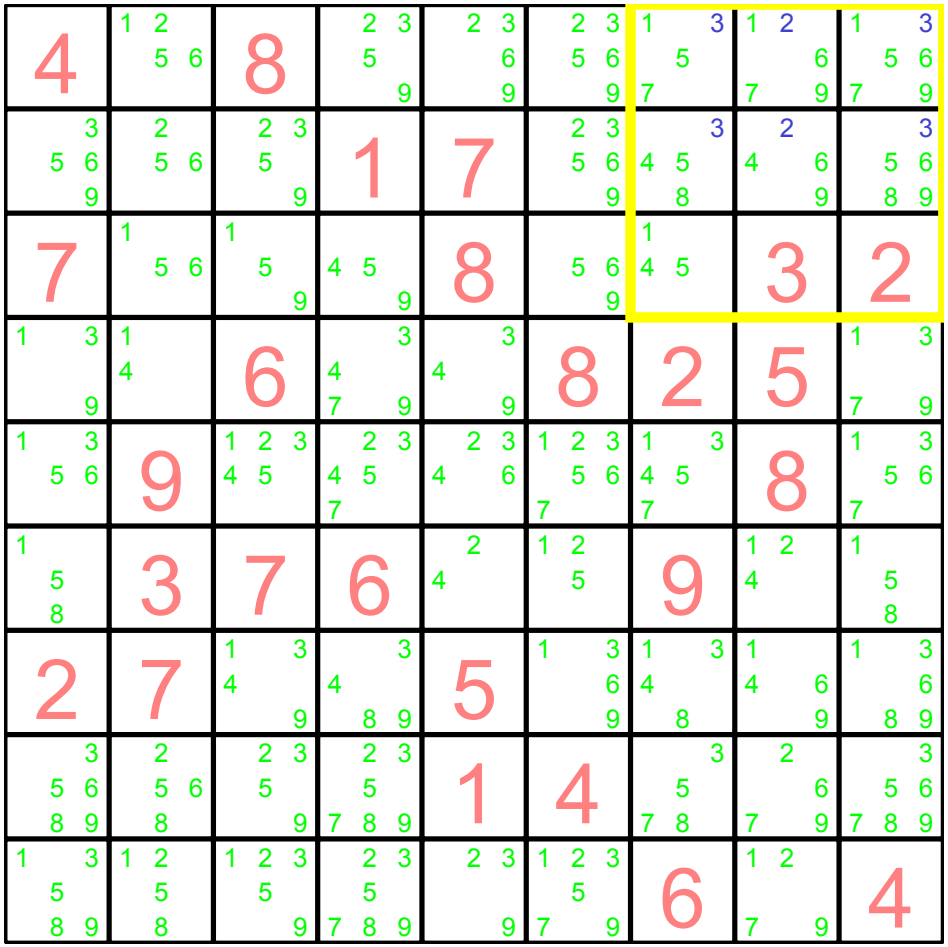
Propagation Steps (Bounds Consistency)



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Propagation Steps (Bounds Consistency)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	2 3 5 6 9	1 5 7	1 6 7 9	1 5 6 7 9
3 5 6 9	2 5 6	2 3 5 9	1	7	2 3 5 6 9	4 5 8	4 6 9	5 6 8 9
7	1 5 6	1 5 9	4 5 9	8	5 6 9	1 4 5	3	2
1	4	6	3 4 7 9	3 4 9	8	2	5	1 3 7 9
5	9	2	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 3 5 6 7
8	3	7	6	2 4	1 2 5	9	1 2 4	1 5 8
2	7	1 3 4 9	3 4 8 9	5	1 3 6 9	1 3 4 8	1 4 6 9	1 3 6 8 9
3 5 6 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	3 5 7 8	2 6 7 9	3 5 6 7 8 9
1 3 5 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 9	1 2 3 5 7 9	6	1 2 7 9	4

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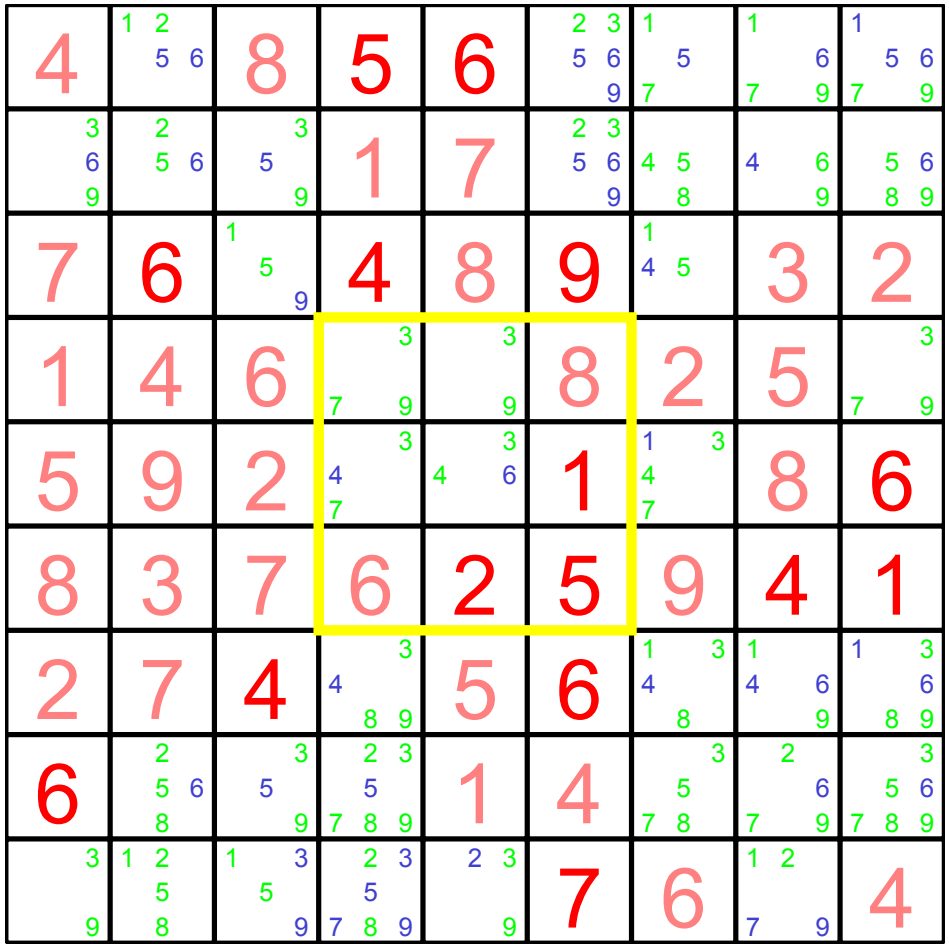
Propagation Steps (Bounds Consistency)



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Propagation Steps (Bounds Consistency)

4	1 2	8	5	6	2 3	1 7	1 7 9	7 9
3 9	2 5	3 9	1	7	2 3	4 5 8	6 9	5 8 9
7	6	1 5	4	8	9	1 5	3	2
1	4	6	3 7 9	3 9	8	2	5	3 7 9
5	9	2	3 7	3 4	1	3 4 7	8	6
8	3	7	6	2	5	9	4	1
2	7	4	3 8 9	5	6	1 3 8	1 9	3 8 9
6	2 5 8	3 9	2 3 7 8 9	1	4	3 5 7 8	2 7 9	3 5 7 8 9
3 9	1 2 5 8	1 5	2 8	3 9	7	6	1 2	4

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Propagation Steps (Bounds Consistency)

4	1 2	8	5	6	2 3	1 7	1 7 9	7 9
3 9	2 5	3 9	1	7	2 3	4 5 8	6 9	5 8 9
7	6	1 5	4	8	9	1 5	3	2
1	4	6	3 7 9	3 9	8	2	5	3 7
5	9	2	3 7	3 4	1	3 7	8	6
8	3	7	6	2	5	9	4	1
2	7	4	3 8 9	5	6	1 3 8	1 9	3 8 9
6	2 5 8	3 9	2 3 7 8 9	1	4	3 5 7 8	2 7 9	3 5 7 8 9
3 9	1 2 5 8	1 5	2 8	3 9	7	6	1 2	4

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Propagation Steps (Bounds Consistency)

4	1 2	8	5	6	2 3	1 7	1 7 9	7 9
3 9	2 5	3 9	1	7	2 3	4 5 8	6 9	5 8 9
7	6	1 5	4	8	9	1 5	3	2
1	4	6	3 7 9	3 9	8	2	5	3 7
5	9	2	3 7	3 4	1	3 7	8	6
8	3	7	6	2	5	9	4	1
2	7	4	3 8 9	5	6	1 3 8	1 9	3 8 9
6	5 8	3 9	2 3 7 8 9	1	4	3 5 7 8	2 7 9	3 5 7 8 9
3 9	1 5 8	1 5	2 8	3 9	7	6	1 2	4

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Propagation Steps (Bounds Consistency)

4	1 2	8	5	6	2 3	1 7	1 7 9	7 9
3 9	2 5	3 9	1	7	2 3	4 5 8	6 9	5 8 9
7	6	1 5	4	8	9	1 5	3	2
1	4	6	3 7 9	3 9	8	2	5	3 7
5	9	2	3 7	3 4	1	3 7	8	6
8	3	7	6	2	5	9	4	1
2	7	4	3 8 9	5	6	1 3 8	1 9	3 8 9
6	5 8	3 9	2 3 8 9	1	4	3 5 7 8	2 7 9	3 5 7 8 9
3 9	1 5 8	1 5	2 8	3 9	7	6	1 2	4

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Propagation Steps (Bounds Consistency)

4	1 2	8	5	6	2 3	1 7	1 7 9	7 9
3 9	2 5	3 9	1	7	2 3	4 5 8	6 9	5 8 9
7	6	1 5	4	8	9	1 5	3	2
1	4	6	3 7 9	3 9	8	2	5	3 7
5	9	2	3 7	3 4	1	3 7	8	6
8	3	7	6	2	5	9	4	1
2	7	4	3 8 9	5	6	1 3 8	1 9	3 8 9
6	5 8	3 9	2 3 8 9	1	4	3 5 7 8	2 7 9	3 5 7 8 9
3 9	1 5 8	1 5	2 8	3 9	7	6	1 2	4

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After Setup (Bounds Consistency)

4	1 2	8	5	6	2 3	1 7	1 7 9	7 9
3 9	2 5	3 9	1	7	2 3	4 5 8	6 9	5 8 9
7	6	1 5	4	8	9	1 5	3	2
1	4	6	3 7 9	3 9	8	2	5	3 7
5	9	2	3 7	3 4	1	3 7	8	6
8	3	7	6	2	5	9	4	1
2	7	4	3 8 9	5	6	1 3 8	1 9	3 8 9
6	5 8	3 9	2 3 8 9	1	4	3 5 7 8	2 7 9	3 5 7 8 9
3 9	1 5 8	1 5	2 8	3 9	7	6	1 2	4

Initial State (Domain Consistency)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

Propagation Steps (Domain Consistency)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

Skip Animation

Propagation Steps (Domain Consistency)

4	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 3 5 6 7 9	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 3 5 6 7 9	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
1 3 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2
1 3 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
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1 3 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 3 5 6 7 9	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 3 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2
1 3 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 6 8 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	2
1 3 5 6 7 8 9	1 2 4 5 6 8	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 6 8 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6	1 4 5 6 7 9	1 4 5 6 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3	2
1 3 5 6 7 8 9	1 2 4 5 6 8	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6	1 4 5 9	1 4 5 6 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3	2
1 3 5 6 7 8 9	1 2 4 5 6 8	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	2	5	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	9	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6	1 4 5 9	1 4 5 6 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3	2
1 3 7 9	1 4	6	1 3 4 7 9	1 3 4 7 9	8	2	5	1 3 4 7 9
1 3 5 6 7 8 9	9	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6	1 4 5 9	4 5 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3	2
1 3 7 9	1 4	6	3 4 7 9	1 3 4 7 9	8	2	5	1 3 4 7 9
1 3 5 6 7 8 9	9	1 2 3 4 5 9	2 3 4 5 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 9	2 3 4 5 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6	1 4 5 9	4 5 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3	2
1 3 7 9	1 4	6	3 4 7 9	1 3 4 7 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	8	1 2 3 4 5 6 7
1 3 5 6 7 8 9	3	7	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 9	2 3 4 5 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6	1 4 5 9	4 5 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3	2
1 3 7 9	1 4	6	3 4 7 9	3 4 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 4 5 6 7	1 2 3 4 5 6 7	8	1 2 3 4 5 6 7
1 3 5 6 7 8 9	3	7	6	2 3 4 6 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
2	7	1 2 3 4 5 9	2 3 4 5 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6	1 4 5 9	4 5 7 9	8	1 4 5 6 7 9	1 4 5 6 7 9	3	2
1 3 7 9	1 4	6	3 4 7 9	3 4 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 4 5 6 7	1 2 3 4 5 6 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 4 5 8	9	1 2 4 5 8	1 2 4 5 8
2	7	1 2 3 4 5 9	2 3 4 5 7 8 9	5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 6 7 9	3	2
1 3 7 9	1 4	6	3 4 7 9	3 4 9	8	2	5	1 3 4 7 9
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1 5 8	3	7	6	2 4	1 2 5	9	1 2 4 5 8	1 2 4 5 8
2	7	1 2 3 4 5 9	2 3 4 5 7 8 9	5	1 2 3 5 6 7 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 7 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 6 7 9	3	2
1 3 7 9	1 4	6	3 4 7 9	3 4 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 2 3 4 5 6 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4 5 8	1 2 4 5 8
2	7	1 3 4 9	3 4 8 9	5	1 3 6 9	1 3 4 6 8 9	1 3 4 6 8 9	1 3 4 6 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	1	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 7 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5 7	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	3 4 5 8	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2
1 3 7 9	1 4	6	3 4 7 9	3 4 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4 5 8	1 2 4 5 8
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1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 7 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5 7	1 2 3 5 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	3 4 5 8	2 3 4 5 6 8 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2
1 3 7 9	1 4	6	3 4 7 9	3 4 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4 5 8	1 2 4 5 8
2	7	1 3 4 9	3 4 8 9	5	1 3 6 9	1 3 4 8	1 3 4 6 8 9	1 3 4 6 8 9
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	3 5 7 8	2 3 5 6 7 8 9	2 3 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 7 9	6	1 2 3 4 5 6 7 8 9	4

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5 7	1 2 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	3 4 5 8	2 4 6 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2
1 3 7 9	1 4	6	3 4 7 9	3 4 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4	1 2 4 5 8
2	7	1 3 4 9	3 4 8 9	5	1 3 6 9	1 3 4 8	1 4 6 9	1 3 4 8 9
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	3 5 7 8	2 6 7 9	2 3 5 6 7 8 9
1 3 5 6 7 8 9	1 2 4 5 6 8	1 2 3 4 5 9	2 3 4 5 7 8 9	2 3 4 6 9	1 2 3 5 6 7 9	6	1 2 4 6 7 9	4

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5 7	1 2 6 7 9	1 2 3 5 6 7 9
3 5 6 8 9	2 4 5 6 8	2 3 4 5 9	1	7	2 3 5 6 9	3 4 5 8	2 4 6 9	2 3 4 5 6 8 9
1 5 6 7 9	1 4 5 6	1 4 5 9	4 5 7 9	8	1 5 6 7 9	1 4 5 7	3	2
1 3 7 9	1 4	6	3 4 7 9	3 4 9	8	2	5	1 3 4 7 9
1 3 5 6 7	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 2 3 4 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4	1 2 4 5 8
2	7	1 3 4 9	3 4 8 9	5	1 3 6 9	1 3 4 8	1 4 6 9	1 3 4 6 8 9
3 5 6 7 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	3 5 7 8	2 6 7 9	2 3 5 6 7 8 9
1 3 5 7 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 9	1 2 3 5 7 9	6	1 2 7 9	4

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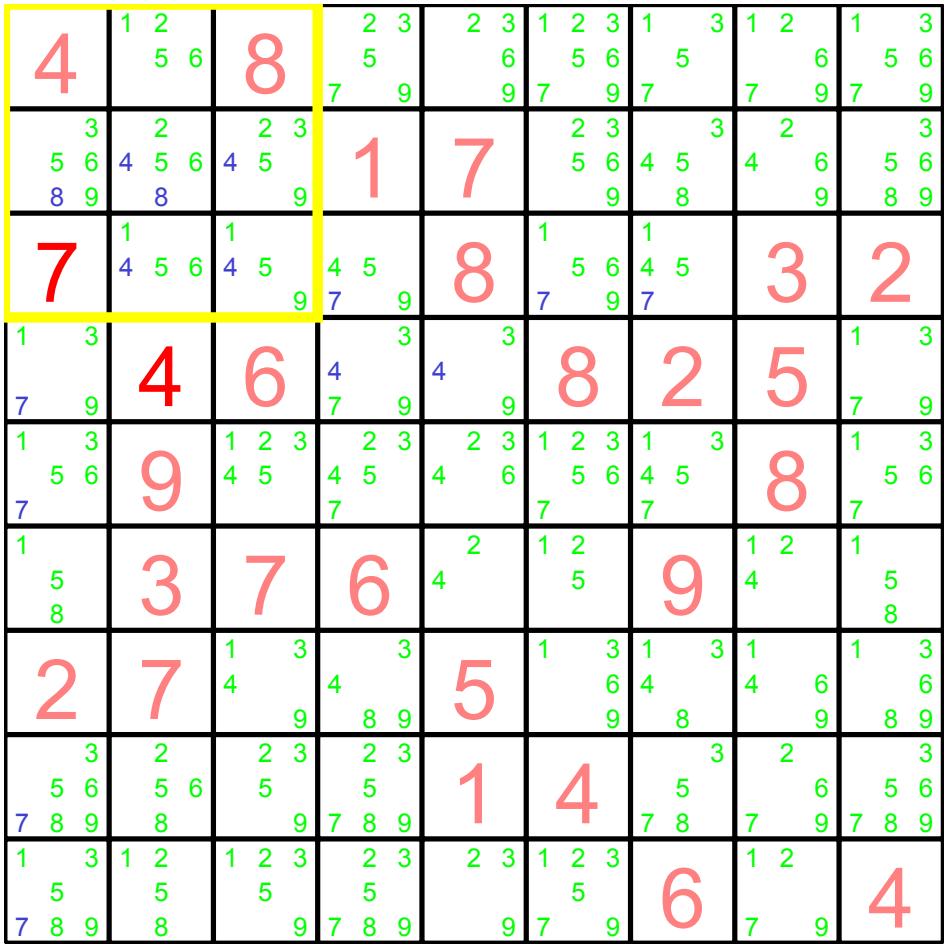
Propagation Steps (Domain Consistency)



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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 7 9	2 3 6 9	1 2 3 5 6 7 9	1 3 5 7	1 2 6 7 9	1 3 5 6 7 9
3 5 6 9	2 5 6	2 3 5 9	1	7	2 3 5 6 9	3 4 5 8	2 4 6 9	3 5 6 8 9
7	1 5 6	1 5 9	4	8	1 5 6 9	1 4 5	3	2
1 3 9	4	6	3 7 9	3 9	8	2	5	1 3 7 9
1 3 5 6	9	1 2 3 4 5	2 3 4 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 3 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4	1 5 8
2	7	1 3 4 9	3 4 8 9	5	1 3 6 9	1 3 4 8	1 4 6 9	1 3 6 8 9
3 5 6 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	3 5 7 8	2 6 7 9	3 5 6 7 8 9
1 3 5 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 9	1 2 3 5 7 9	6	1 2 7 9	4

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Propagation Steps (Domain Consistency)

4	1 2 5 6	8	2 3 5 9	2 3 6 9	2 3 5 6 9	1 3 5 7	1 2 6 7 9	1 3 5 6 7 9
3 5 6 9	2 5 6	2 3 5 9	1	7	2 3 5 6 9	3 4 5 8	2 4 6 9	3 5 6 8 9
7	1 5 6	1 5 9	4	8	5 6 9	1 5	3	2
1 3 9	4	6	3 7 9	3 9	8	2	5	1 3 7 9
1 3 5 6	9	1 2 3 4 5	2 3 5 7	2 3 4 6	1 2 3 5 6 7	1 3 4 5 7	8	1 3 5 6 7
1 5 8	3	7	6	2 4	1 2 5	9	1 2 4	1 5 8
2	7	1 3 4 9	3 8 9	5	1 3 6 9	1 3 4 8	1 4 6 9	1 3 6 8 9
3 5 6 8 9	2 5 6 8	2 3 5 9	2 3 5 7 8 9	1	4	3 5 7 8	2 6 7 9	3 5 6 7 8 9
1 3 5 8 9	1 2 5 8	1 2 3 5 9	2 3 5 7 8 9	2 3 9	1 2 3 5 7 9	6	1 2 7 9	4

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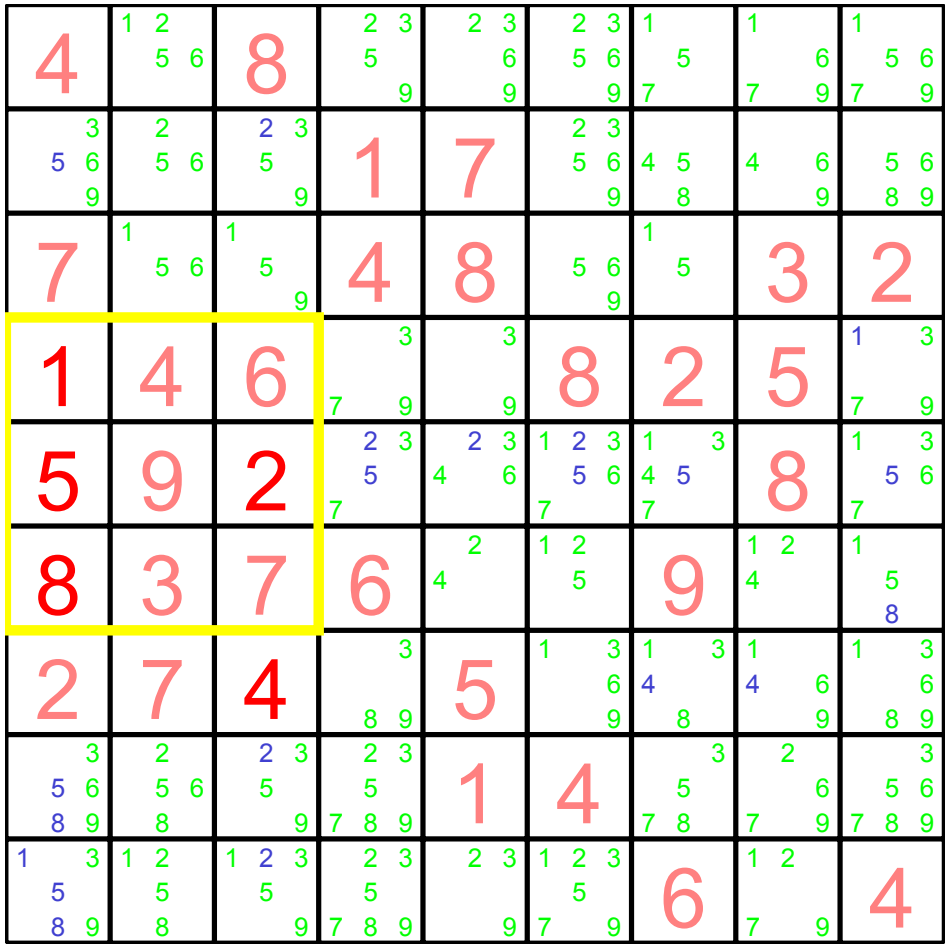
Propagation Steps (Domain Consistency)



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Propagation Steps (Domain Consistency)

4	2	8	5	6	3	1	1 6 7 9	1 5 6 7 9
3 6 9	5	3 5 9	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	3 7 9	3 9	8	2	5	3 7 9
5	9	2	3 7	4	1	1 3 4 7	8	6
8	3	7	6	2	5	9	4	1
2	7	4	3 8 9	5	6	8	1	1 3 6 8 9
6	8	3 5 9	2	1	4	3 5 7 8	2 6 7 9	5
3 9	1	5	8	2 3 9	7	6	2	4

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Propagation Steps (Domain Consistency)

4	2	8	5	6	3	1	7 9	7 9
3 9	5	3 9	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	3 7 9	3 9	8	2	5	3 7 9
5	9	2	3 7	4	1	3 7	8	6
8	3	7	6	2	5	9	4	1
2	7	4	3 9	5	6	8	1	3 9
6	8	3 9	2	1	4	3 7	7 9	5
3 9	1	5	8	3 9	7	6	2	4

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Propagation Steps (Domain Consistency)

4	2	8	5	6	3	1	7 9	7 9
3 9	5	3 9	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	3 7 9	3 9	8	2	5	3 7
5	9	2	3 7	4	1	3 7	8	6
8	3	7	6	2	5	9	4	1
2	7	4	3 9	5	6	8	1	3 9
6	8	3 9	2	1	4	3 7	7 9	5
3 9	1	5	8	3 9	7	6	2	4

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Propagation Steps (Domain Consistency)

4	2	8	5	6	3	1	7 9	7 9
3 9	5	3 9	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	3 7 9	3 9	8	2	5	3 7
5	9	2	3 7	4	1	3 7	8	6
8	3	7	6	2	5	9	4	1
2	7	4	3 9	5	6	8	1	3 9
6	8	3 9	2	1	4	3 7	7 9	5
3 9	1	5	8	3 9	7	6	2	4

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Propagation Steps (Domain Consistency)

4	2	8	5	6	3	1	7 9	7 9
3 9	5	3 9	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	3 7 9	3 9	8	2	5	3 7
5	9	2	3 7	4	1	3 7	8	6
8	3	7	6	2	5	9	4	1
2	7	4	3 9	5	6	8	1	3 9
6	8	3 9	2	1	4	3 7	7 9	5
3 9	1	5	8	3 9	7	6	2	4

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Propagation Steps (Domain Consistency)

4	2	8	5	6	3	1	7 9	7 9
3 9	5	3 9	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	3 7 9	3 9	8	2	5	3 7
5	9	2	3 7	4	1	3 7	8	6
8	3	7	6	2	5	9	4	1
2	7	4	3 9	5	6	8	1	3 9
6	8	3 9	2	1	4	3 7	7 9	5
3 9	1	5	8	3 9	7	6	2	4

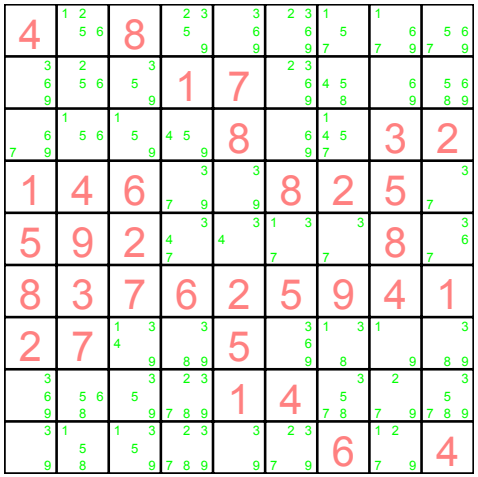
Back to Start

After Setup (Domain Consistency)

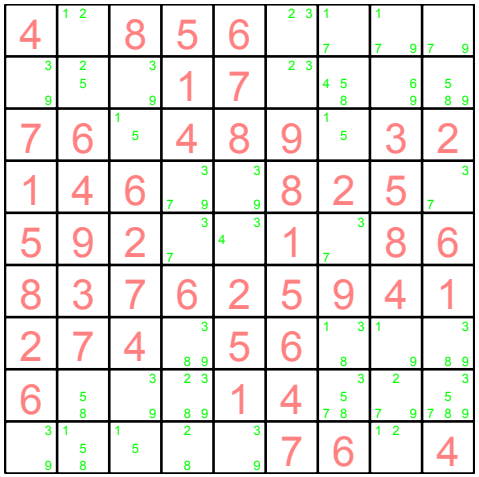
4	2	8	5	6	3	1	7 9	7 9
3 9	5	3 9	1	7	2	4	6	8
7	6	1	4	8	9	5	3	2
1	4	6	3 7 9	3 9	8	2	5	3 7
5	9	2	3 7	4	1	3 7	8	6
8	3	7	6	2	5	9	4	1
2	7	4	3 9	5	6	8	1	3 9
6	8	3 9	2	1	4	3 7	7 9	5
3 9	1	5	8	3 9	7	6	2	4

Comparison

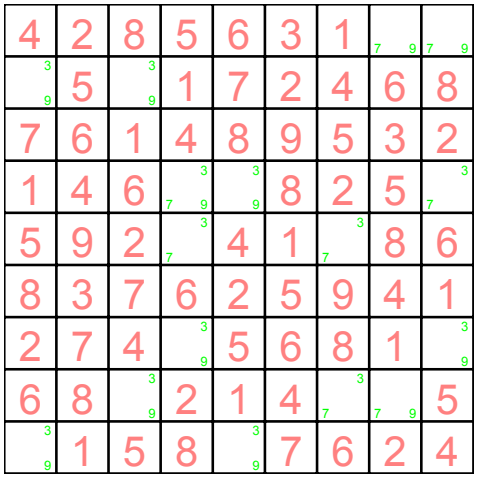
Forward Checking



Bounds Consistency



Domain Consistency



Typical?

- This does not always happen
- Sometimes, two methods produce same amount of propagation
- Possible to predict in certain special cases
- In general, tradeoff between speed and propagation
- Not always fastest to remove inconsistent values early
- But often required to find a solution at all

Outline

Constraint Propagation

2) What are the implications of the current variable assignments for the other unassigned variables?

Definition (Domain consistency)

A constraint C on the variables $X_1, \ldots, X_k$ is called domain consistent if for each variable X_i and each value $v_i \in D(X_i)$ $(i = 1, \ldots, k)$, there exists a value $v_j \in D(X_j)$ for all $j \neq i$ such that $(d_1, \ldots, d_k) \in C$.

Loose definition

Domain filtering is the removal of values from variable domains that are not consistent with an individual constraint.

Constraint propagation is the repeated application of all domain filtering of individual constraints until no domanin reduction is possible anymore.

Outline

In Gecode: http://www.gecode.org/doc-latest/reference/group__TaskModelInt.html

In Minizinc: from the root of the minizinc installation:

```
lib/minizinc/std/globals.mzn
gnome-open doc/index.html
```

Integer Variables


```
IntVar x(home, 1, 4);
IntVar y(x);
```

```
IntVar x(home, 1, 4);
IntVar y;
y=x;
```

In both cases y is not allocating a new data structure but it is a reference to the data structure of x

Overloaded operator:

```
std::cout << x <<std::endl;
```

Access the domain of the variables via iterator:

```
for (IntVarValues i(x); i(); ++i)
std::cout << i.val() << ' ';
```

Access the ranges via iterator:

```
for (IntVarRanges i(x); i(); ++i)
std::cout << i.min() << ".." << i.max() << ' ';
```

Variable Interface

`assigned()`, `update()`

```
IntVar x(home, 0, 0);
rel(home, x, IRT_NQ, 0);
home.status();
```

Variables never reach empty domains, not either when the status is failed.
`status()` can be good for debugging purposes: check at the root node

```
int main(int argc, char* argv[]) {
   Options opt("SEND+MORE=MONEY");
   opt.parse(argc, argv);
   Money* m = new Money(opt);
   SpaceStatus status = m->status();
   if (status == SS_FAILED)
    cout << "Status: " << m->status() << " the space is failed at root."<< endl;
   else if (status == SS_SOLVED)
    cout << "Status: " << m->status()
    << " the space is not failed but the space has no brancher left."<< endl;
   else if (status == SS_BRANCH)
    cout << "Status: " << m->status()
    << " the space is not failed and we need to start branching."<< endl;
   m->print(cout);
   DFS<Money> e(m);
   while (Money* s = e.next()) {
    s->print(cout);
    delete s;
   }
   delete m;
   return 0;}
```

Arrays of Variables


```
IntVarArray x(home, 4, -10, 10);
```

```
IntVarArray x(home, 4); // does not create the array
for (int i=0; i<4; i++)
x[i] = IntVar(home, -10, 10);
```

Variables are only deleted when the space is deleted.

Matrix Interface


```
IntVarArgs x(n*m);

Matrix<IntVarArgs> mat(x, n, m);

IntVar mij = mat(i,j);
```

Argument Arrays

For:

- dynamically builded arrays
- temporary variables
- arguments for post functions

They allocate memory from the heap and the memory is freed when their desctructor is executed. (They cannot be updated.)

```
IntVarArgs x;
IntVarArgs x(5);
IntVarArgs x(home,5,0,10);
```

```
IntVarArgs x;
x << IntVar(home,0,10);
IntVarArgs y;
y << IntVar(home,10,20);
y << x;
linear(home, IntVarArgs()<<x[0]<<x[1], IRT_EQ, 0);
```

Concatenation:

```
IntVarArgs z = x+y;
```

Slices


```
IntVarArgs x(home, 10, 0, 10);
```

```
x.slice(5) // returns an array with elements x[5],x[6], . . . ,x[9]
x.slice(5,1,3) // returns x[5],x[6],x[7]
x.slice(5,-1) // returns x[5],x[4], . . . ,x[0]
x.slice(3,3) // returns x[3],x[6],x[9] .
x.slice(8,-2) // returns x[8],x[6],x[4],x[2],x[0]
x.slice(8,-2,3) // returns x[8],x[6],x[4]
```

```
IntArgs::create(n,start,inc)
```