DM841
DISCRETE OPTIMIZATION

Exercises
Modelling in CP

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Generalized Nurse Scheduling

Same problem as in the lectures but with number of nurses much larger than the number of shifts. Moreover coverage constraints, requiring a number of nurses in each shift.

- One of the two views does not work here anymore.
- Coverage constraint can be handled by cardinality constraint

Zebra

http://en.wikipedia.org/wiki/Zebra_Puzzle

A street has five differently colored houses on it. Five men of different nationalities live in these five houses. Each man smokes a different brand of American cigarettes, each man likes a different drink, and each has a different pet animal.

1. The Englishman lives in the red house.
2. The Spaniard owns the dog.
3. Coffee is drunk in the green house.
4. The Ukrainian drinks tea.
5. The green house is immediately to the right of the ivory house.
6. The Old Gold smoker owns snails.
7. Kools are smoked in the yellow house.
8. Milk is drunk in the middle house.
9. The Norwegian lives in the first house.
10. The man who smokes Chesterfields lives in the house next to the man with the fox.
11. Kools are smoked in the house next to a house where the horse is kept.
12. The Lucky Strike smoker drinks orange juice.
13. The Japanese smokes Parliaments.
14. The Norwegian lives next to the blue house.

Now, who drinks water? Who owns the zebra?

Model

Variables: 25:

- nationality: englishman, spaniard, ukrainian, japanese, Norwegian
- pet: dog, snails, fox, horse, (zebra)
- cigarettes: Kools, Lucky Strike, Parliaments, Chesterfields, Old Gold
- drink: the, cafe, milk, juice, (water)
- color: red, green, ivory, yellow, blue.

Domains: [1..5]

Constraints

all_different(Englishman, Spaniard, Ukrainian, Japanese, Norwegian)
all_different(dog, snails, fox, horse, zebra)
all_different(Kools, Lucky Strike, Parliaments, Chesterfields, Old Gold)
all_different(the, caffe, milk, juice, water)
all_different(red, green, ivory, yellow, blue)

1. The Englishman lives in the red house.
   Englishman=red
2. The Spaniard owns the dog.
   Spaniard=dog
3. Coffee is drunk in the green house. coffee=green
4. The Ukrainian drinks tea.
   Ukrainian=tea
5. The green house is immediately to the right of the ivory house.
   green = ivory + 1
6. The Old Gold smoker owns snails.
   Old Gold = snails
7. Kools are smoked in the yellow house.
   Kools=yellow
8. Milk is drunk in the middle house.
   milk=3
9. The Norwegian lives in the first house.
   Norwegian=1

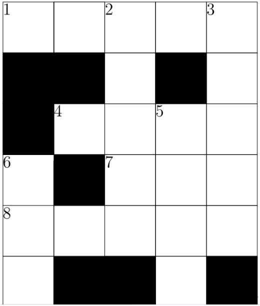
10. The man who smokes Chesterfields lives in the house next to the man with the fox.
    |Chesterfields-fox|=1
11. Kools are smoked in the house next to a house where the horse is kept.
    Kools=horse+1
11. The Lucky Strike smoker drinks orange juice.
    Lucky Strike=juice
12. The Japanese smokes Parliaments.
    Japanese=Parliaments
13. The Norwegian lives next to the blue house.
    |Norwegian-blue|=1

Crosswords

Symbolic constraint satisfaction problems [MPG ch 21]

Consider the crossword grid of the figure and suppose we are to fill it with the words taken from the following list:

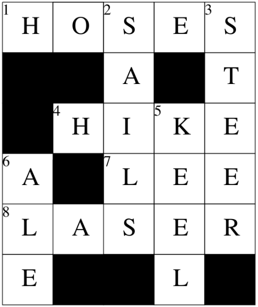
- HOSES, LASER, SAILS, SHEET, STEER,
- HEEL, HIKE, KEEL, KNOT, LINE,
- AFT, ALE, EEL, LEE, TIE.



Formulate the problem as a CSP.
Is the initial status of the formulated CSP arc consistent? If not, enforce arc consistency.

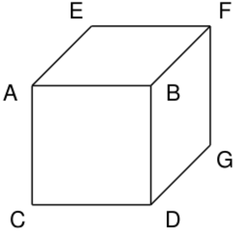
Variables: $x_1, \ldots, x_8$
Domains: $x_6 \in \{AFT, ALE, EEL, LEE, TIE\}$, ecc.
Constraints: a constraint for each crossing. For positions 1 and 2:

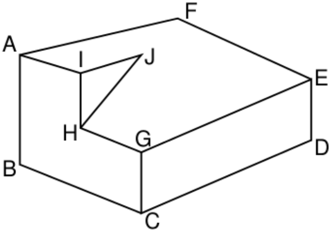
$$\begin{aligned} C_{1,2} := \{ & (HOSES, SAILS), (HOSES, SHEET), \\ & (HOSES, STEER), (LASER, SAILS), \\ & (LASER, SHEET), (LASER, STEER)\}. \end{aligned}$$



In Gecode: `extensional`

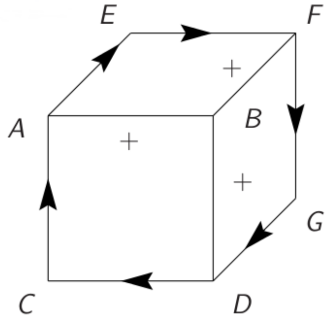
Do this drawnings represent feasible 3D objects?



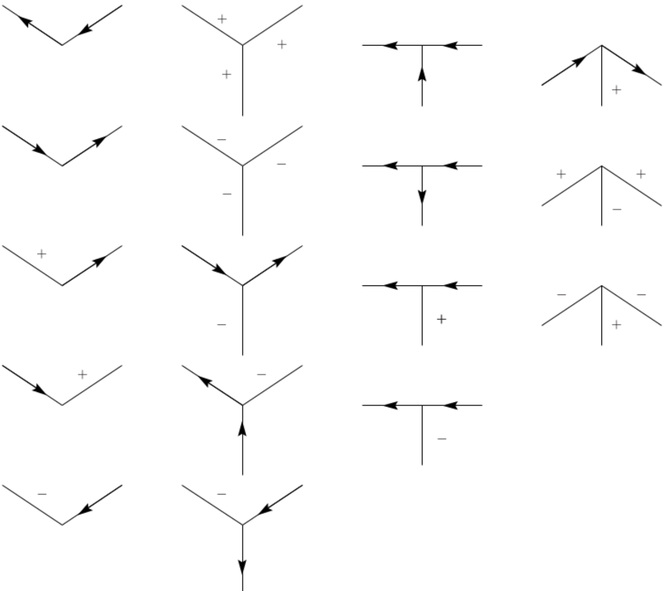


Labeling of edges:

- $+$ to mark the convex edges
  (270 degrees to rotate a plane over the other through the viewer)
- $-$ to mark the concave edges
  (90 degrees to rotate a plane over the other through the viewer)
- arrows to mark the boundary edges
  (orientation such that scene is on right-hand side)



Legal junctions



Is there a labeling of edges in such a way that only labeled junctions listed in the figure exist?

Model 1

Variables: junctions: 4 variables L, fork, T, arrow.

Domains: the good labellings from the columns of figure in previous slide.

To represent label-ling in textual form, use translation tables:

$$L \in \{(\rightarrow, \leftarrow), (\leftarrow, \rightarrow), (+ \leftarrow), (\leftarrow, +), (- \leftarrow), (\rightarrow, -)\}$$

Constraints: junctions share edges:

Example for the cube:

Junctions A and B share edge AB, hence limits on the values used for junctions A and B (like in the crosswords example)

$$C_{AE} = \{((\leftarrow, \rightarrow, +), (\rightarrow, \leftarrow)), ((\leftarrow, \rightarrow, +), (-, \leftarrow)) \\ ((+, +, -), (\leftarrow, +)), ((-, -, +), (\rightarrow, -))\}$$

Model 2

Variables: edges

Domains: $\{+, - \leftarrow, \rightarrow\}$

Constraints: junctions

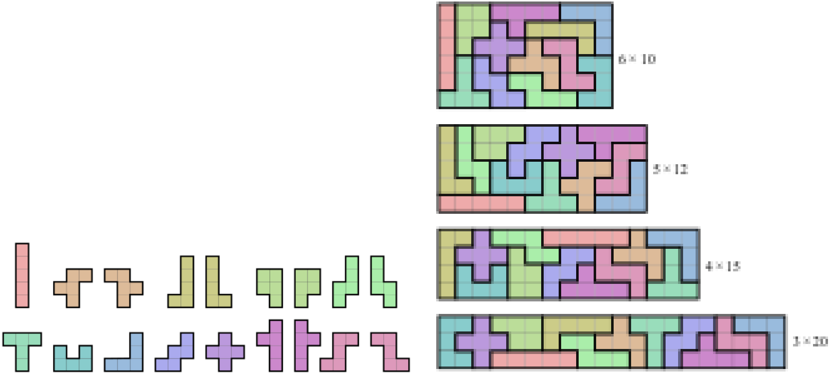
Four types of constraints: L, fork, T and arrow.

Example:

$$
\begin{aligned}
L :=\{&(\rightarrow, \leftarrow), (\leftarrow, \rightarrow), (+, \rightarrow),\\
&(\leftarrow, +), (-, \leftarrow), (\rightarrow, -)\}.
\end{aligned}
$$

The cube as CSP:
arrow(AC, AE, AB),
fork(BA, BF, BD),
L(CA, CD),
arrow (DG, DC , DB),
L(EF, EA),
arrow(FE, FG, FB),
L(GD, GF).

Pentominoes, Nonograms, Battleships



By R. A. Nonenmacher - Created by me, GFDL, https://commons.wikimedia.org/w/index.php?curid=4416149

Pentominoes, Nonograms, Battleships

[MPG, ch 16]

		2	2	2	2	2	2	2	
	3	3	2	2	2	2	2	3	3
2 2									
4 4									
1 3 1									
2 1 2									
1 1									
2 2									
2 2									
3									
1									

Figure 16.1: Example nonogram puzzle

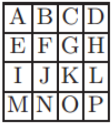
		2	2	2	2	2	2	2	
	3	3	2	2	2	2	2	3	3
2 2		■	■				■	■	
4 4	■	■	■	■		■	■	■	■
1 3 1	■			■	■	■			■
2 1 2	■	■			■			■	■
1 1		■						■	
2 2		■	■				■	■	
2 2			■	■		■	■		
3				■	■	■			
1					■				

Figure 16.2: Solution to the example puzzle

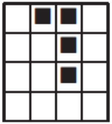
Pentominoes, Nonograms, Battleships

[Lagerkvist and Pesant, 2008]

(a) Shape to place



(b) Grid to place in



(c) Possible placement of shape.

Use of `regular` constraints

References

Lagerkvist M.Z. and Pesant G. (2008). **Modeling irregular shape placement problems with regular constraints**. In *First Workshop on Bin Packing and Placement Constraints BPPC'08*.