DM841

Discrete Optimization

Part I

Lecture 4

Introduction to Gecode

Marco Chiarandini

Department of Mathematics & Computer Science
University of Southern Denmark

Outline

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Resume

- CP modeling examples
  - Graph labeling with consecutive numbers
  - Send More Money
- Constraint programming:
  representation (modeling language) + reasoning (propagation + search)
  - model
  - propagate, filtering, pruning
  - search = backtracking + branching
- Gecode: model in Script class implementation
  - Variables:
    declare as members
    initialize in constructor
    update in copy constructor
  - Posting constraints (in constructor)
  - Create branching (in constructor)
  - Provide copy constructor (recomputation) and copy function (cloning)

Constraint Programming with Gecode

Overview

- Program model as ***script***
  - declare variables
  - post constraints (creates propagators)
  - define branching
- Solve script
  - basic search strategy
  - Gist: interactive visual search

Program Model as Script

Script: Overview

- Script is class inheriting from class Space
  - members store variables regarded as solution
- Script constructor
  - initialize variables
  - post propagators for constraints
  - define branching
- Copy constructor and copy function
  - copy a Script object during search
- Exploration takes Script object as input
  - returns object representing solution
- Main function
  - invokes search engine

Script for SMM: Structure

```
#include <gecode/int.hh>
#include <gecode/search.hh>

using namespace Gecode;

class SendMoreMoney : public Space {
protected:
  IntVarArray l; // Digits for the letters
public:
  // Constructor for script
  SendMoreMoney(void) … { … }
  // Constructor for cloning
  SendMoreMoney(bool share, SendMoreMoney& s) … { … }
  // Perform copying during cloning
  virtual Space* copy(bool share) { … }
  // Print solution
  void print(void) { … }
};

…
```

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#include <gecode/sear

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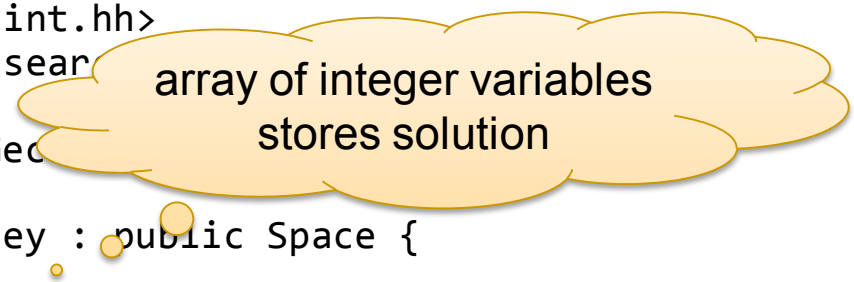
  virtual Space* copy(bool share) { … }
  // Print solution
  void print(void) { … }
};

…
```



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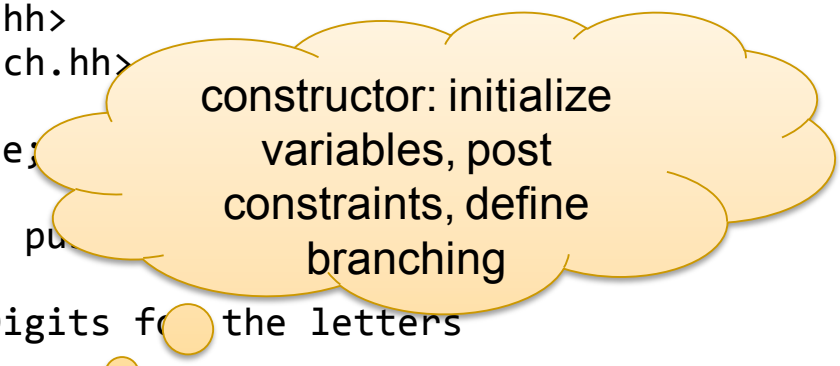
  // Perform copying during cloning
  virtual Space* copy(bool share) { … }
  // Print solution
  void print(void) { … }
};

…
```



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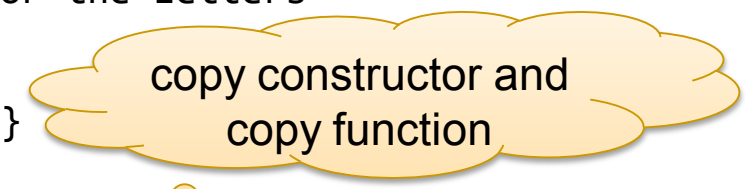
  void print(void) { … }
};

…
```



Script for SMM: Constructor

```
SendMoreMoney(void) : l(*this, 8, 0, 9) {
  IntVar s(l[0]), e(l[1]), n(l[2]), d(l[3]),
         m(l[4]), o(l[5]), r(l[6]), y(l[7]);
  // Post constraints
  …
  // Post branchings
  …
}
```

Script for SMM: Constructor

```
SendMoreMoney(void) : l(*this, 8, 0, 9) {
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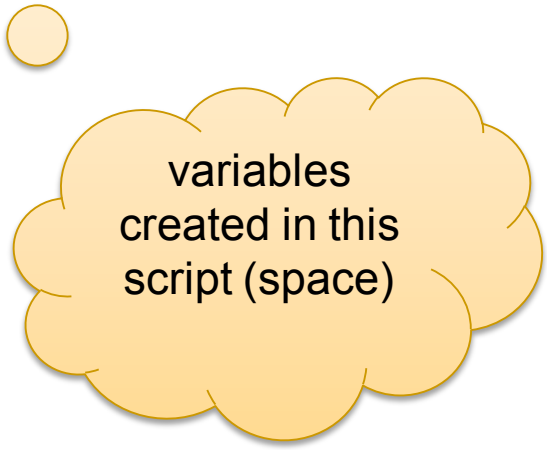
  // Post constraints
  …
  // Post branchings
  …
}
```



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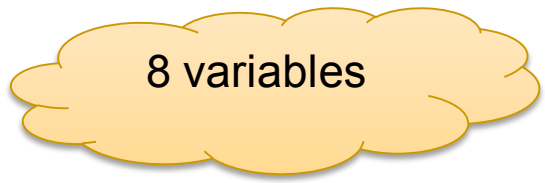
  // Post constraints
  …
  // Post branchings
  …
}
```



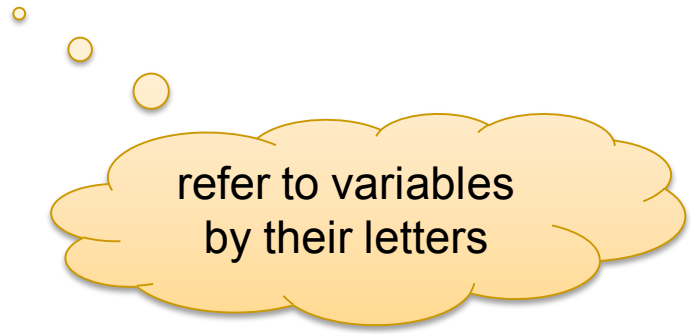
Script for SMM: Constructor

```
SendMoreMoney(void) : l(*this, 8, 0, 9) {
  IntVar s(l[0]), e(l[1]), n(l[2]), d(l[3]),
         m(l[4]), o(l[5]), r(l[6]), y(l[7]);
  // Post constraints
  …
  // Post branchings
  …
}
```

take values between 0 and 9 (digits)

Script for SMM: Constructor

```
SendMoreMoney(void) : l(*this, 8, 0, 9) {
  IntVar s(l[0]), e(l[1]), n(l[2]), d(l[3]),
         m(l[4]), o(l[5]), r(l[6]), y(l[7]);
  // Post constraints
  …
  // Post branchings
  …
}
```



Script for SMM: Constructor

```
SendMoreMoney(void) : l(*this, 8, 0, 9) {
  IntVar s(l[0]), e(l[1]), n(l[2]), d(l[3]),
         m(l[4]), o(l[5]), r(l[6]), y(l[7]);
  // No leading zeros (IRT: integer relation type)
  rel(*this, s, IRT_NQ, 0);
  rel(*this, m, IRT_NQ, 0);
  // All letters must take distinct digits
  distinct(*this, l);
  // The linear equation must hold
  …
  // Branch over the letters
  …
}
```

Posting Constraints

- Defined in namespace `Gecode`
- Check documentation for available constraints
- Take script reference as first argument
  - where is the propagator for the constraint to be posted!
  - script is a subclass of Space (computation space)

Linear Equations and Linear Constraints

- Equations of the form

  $c_1 \cdot x_1 + \ldots + c_n \cdot x_n = d$

  - integer constants: c_i and d
  - integer variables: x_i
- In Gecode specified by arrays
  - integers (`IntArgs`) c_i
  - variables (`IntVarArray, IntVarArgs`) x_i
- Not only equations
  - `IRT_EQ, IRT_NQ, IRT_LE, IRT_GR, IRT_LQ, IRT_GQ`
  - equality, disequality, inequality (less, greater, less or equal, greater or equal)

Script for SMM: Constructor

```
SendMoreMoney(void) : l(*this, 8, 0, 9) {
  …
  // The linear equation must hold
  IntArgs c(4+4+5); IntVarArgs x(4+4+5);
  c[0]=1000; c[1]=100; c[2]=10; c[3]=1;
  x[0]=s;    x[1]=e;   x[2]=n;  x[3]=d;
  c[4]=1000; c[5]=100; c[6]=10; c[7]=1;
  x[4]=m;    x[5]=o;   x[6]=r;  x[7]=e;
  c[8]=-10000; c[9]=-1000; c[10]=-100; c[11]=-10; c[12]=-1;
  x[8]=m;      x[9]=o;     x[10]=n;    x[11]=e;   x[12]=y;
  linear(*this, c, x, IRT_EQ, 0);
  // Branch over the letters
  …
}
```

Linear Expressions

- Other options for posting linear constraints are available: minimodeling support
  - linear expressions
  - Boolean expressions
  - matrix classes
  - …
- See the examples that come with Gecode

Script for SMM: Constructor

```
…
#include <gecode/minimodel.hh>
…

  SendMoreMoney(void) : l(*this, 8, 0, 9) {
    …
    // The linear equation must hold
    post(*this,              1000*s + 100*e + 10*n + d
                           + 1000*m + 100*o + 10*r + e
               == 10000*m + 1000*o + 100*n + 10*e + y);
    // Branch over the letters
    …
  }
```

Script for SMM: Constructor

```
SendMoreMoney(void) : l(*this, 8, 0, 9) {
  …
  // Branch over the letters
  branch(*this, l, INT_VAR_SIZE_MIN, INT_VAL_MIN);
}
```

Branching

- Which variable to choose
  - given order `INT_VAR_NONE`
  - smallest size `INT_VAR_SIZE_MIN`
  - smallest minimum `INT_VAR_MIN_MIN`
  - …
- How to branch: which value to choose
  - try smallest value `INT_VAL_MIN`
  - split (lower first) `INT_VAL_SPLIT_MIN`
  - …

Script for SMM: Copying

```
// Constructor for cloning
SendMoreMoney(bool share, SendMoreMoney& s) : Space(share, s) {
  l.update(*this, share, s.l);
}
// Perform copying during cloning
virtual Space* copy(bool share) {
  return new SendMoreMoney(share,*this);
}
```

Script for SMM: Copying

```
// Constructor for cloning
SendMoreMoney(bool share, SendMoreMoney& s) : Space(share, s) {
  l.update(*this, share, s.l);
}
// Perform copying during cloning
virtual Space* copy(bool share) {
  return new SendMoreMoney(share,*th
}
```

update all variables needed for solution

Script for SMM: Copying

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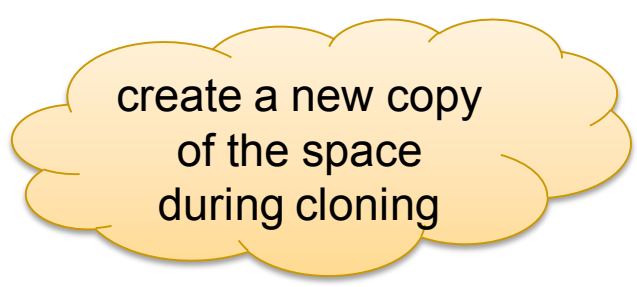
// Constructor for cloning
SendMoreMoney(bool share, SendMoreMoney& s) : Space(share, s) {
  l.update(*this, share, s.l);
}
// Perform copying during cloning
virtual Space* copy(bool share) {
  return new SendMoreMoney(share,*this);
}
```



Copying

- Required during exploration
  - before starting to guess: make copy
  - when guess is wrong: use copy
  - discussed later
- Copy constructor and copy function needed
  - copy constructor is specific to script
  - updates (copies) variables in particular

Copy Constructor And Copy Function

- Always same structure
- Important!
  - must update the variables of a script!
  - if you forget: crash, boom, bang, …

Script for SMM: Print Function

```
…
  // Print solution
  void print(void) {
    std::cout << l << std::endl;
  }
```

Summary: Script

- Variables
  - declare as members
  - initialize in constructor
  - update in copy constructor
- Posting constraints
- Create branching
- Provide copy constructor and copy function

Solving Scripts

Available Search Engines

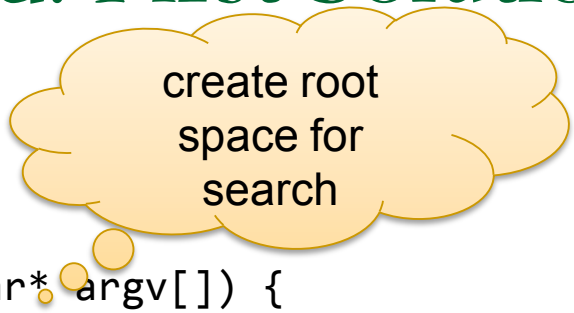
- Returning solutions one by one for script
  - `DFS` depth-first search
  - `BAB` branch-and-bound
  - `Restart, LDS`
- Interactive, visual search
  - `Gist`

Main Method: First Solution

```
…

int main(int argc, char* argv[]) {
  SendMoreMoney* m = new SendMoreMoney;
  DFS<SendMoreMoney> e(m);
  delete m;
  if (SendMoreMoney* s = e.next()) {
    s->print(); delete s;
  }
  return 0;
}
```

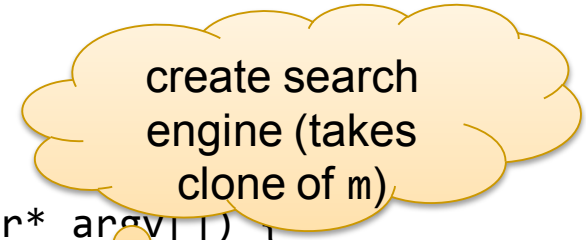
Main Method: First Solution



…

```
int main(int argc, char* argv[]) {
  SendMoreMoney* m = new SendMoreMoney;
  DFS<SendMoreMoney> e(m);
  delete m;
  if (SendMoreMoney* s = e.next()) {
    s->print(); delete s;
  }
  return 0;
}
```

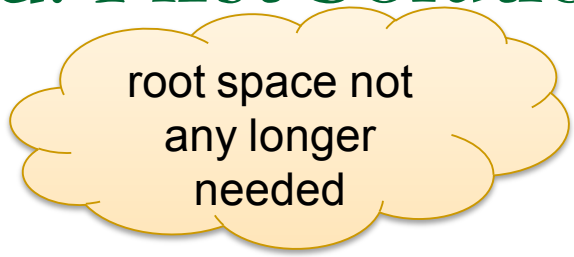
Main Method: First Solution




```
…

int main(int argc, char* argv[]) {
  SendMoreMoney* m = new SendMoreMoney;
  DFS<SendMoreMoney> e(m);
  delete m;
  if (SendMoreMoney* s = e.next()) {
    s->print(); delete s;
  }
  return 0;
}
```

Main Method: First Solution



…

```
int main(int argc, char* argv[]) {
  SendMoreMoney* m = new SendMoreMoney;
  DFS<SendMoreMoney> e(m);
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    s->print(); delete s;
  }
  return 0;
}
```

Main Method: First Solution

```
…

int main(int argc, char* argv[
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  DFS<SendMoreMoney> e(m);
  delete m;
  if (SendMoreMoney* s = e.next()) {
    s->print(); delete s;
  }
  return 0;
}
```

search first solution and print it

Main Method: All Solutions

```
…

int main(int argc, char* argv[]) {
  SendMoreMoney* m = new SendMoreMoney;
  DFS<SendMoreMoney> e(m);
  delete m;
  while (SendMoreMoney* s = e.next()) {
    s->print(); delete s;
  }
  return 0;
}
```

Gecode Gist

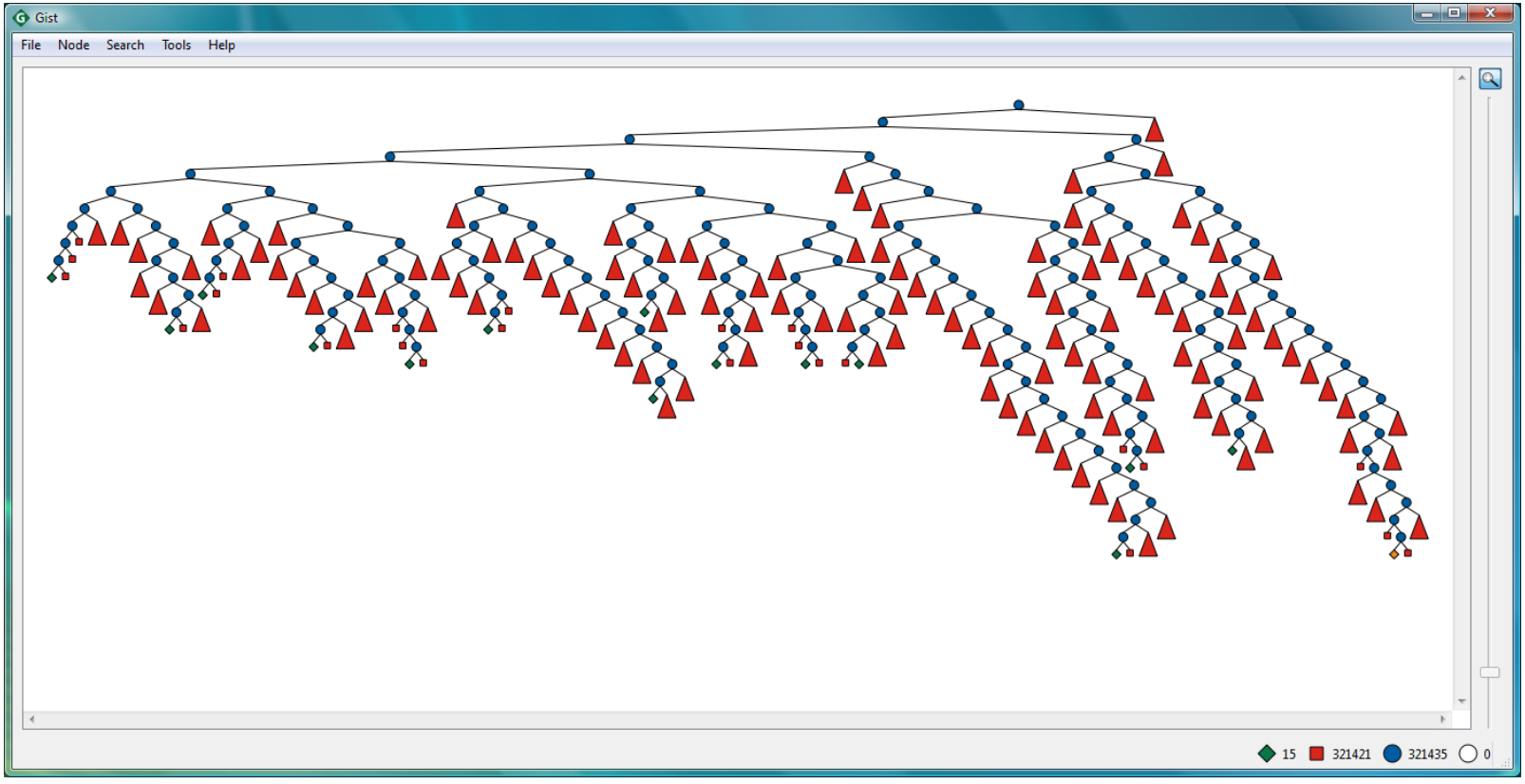
- A graphical tool for exploring the search tree
  - explore tree step by step
  - tree can be scaled
  - double-clicking node prints information: inspection
  - search for next solution, all solutions
  - …
- Best to play a little bit by yourself
  - hide and unhide failed subtrees
  - …

Main Function: Gist

```
#include <gecode/gist.hh>

int main(int argc, char* argv[]) {
  SendMoreMoney* m = new SendMoreMoney;
  Gist::dfs(m);
  delete m;
  return 0;
}
```

Gist Screenshot



Best Solution Search

Reminder: SMM++

- Find distinct digits for letters, such that

```
  SEND
+  MOST
-------
= MONEY
```

and `MONEY` maximal

Script for SMM++

- Similar, please try it yourself at home
- In the following, referred to by `SendMostMoney`

Solving SMM++: Order

- Principle
  - for each solution found, constrain remaining search for better solution
- Implemented as additional method

```
virtual void constrain(const Space& b) {
  …
}
```

- Argument `b` refers to so far best solution
  - only take values from `b`
  - never mix variables!
- Invoked on object to be constrained

Order for SMM++

```
virtual void constrain(const Space& _b) {
  const SendMostMoney& b =
    static_cast<const SendMostMoney&>(_b);

  IntVar e(l[1]), n(l[2]), m(l[4]), o(l[5]), y(l[8]);

  IntVar b_e(b.l[1]), b_n(b.l[2]), b_m(b.l[4]),
         b_o(b.l[5]), b_y(b.l[8]);

  int money = (10000*b_m.val()+1000*b_o.val()+100*b_n.val()+
               10*b_e.val()+b_y.val());

  post(*this, 10000*m+1000*o+100*n+10*e+y > money);
}
```

rel

value of any next solution

value of current best solution b

Main Method: All Solutions

```
…

int main(int argc, char* argv[]) {
  SendMostMoney* m = new SendMostMoney;
  BAB<SendMostMoney> e(m);
  delete m;
  while (SendMostMoney* s = e.next()) {
    s->print(); delete s;
  }
  return 0;
}
```

Main Function: Gist

```
#include <gecode/gist.hh>

int main(int argc, char* argv[]) {
  SendMostMoney* m = new SendMostMoney;
  Gist::bab(m);
  delete m;
  return 0;
}
```

Summary: Solving

- Result-only search engines
  - `DFS, BAB`
- Interactive search engine
  - `Gist`

- Best solution search uses constrain-method for posting constraint
- Search engine independent of script and constrain-method

Exercise

- Solve in Gecode the problem:

  $$\texttt{send} + \texttt{more} = \texttt{money}$$

  What is the solution that maximizes money? How many solutions are there for the decision version? Compare using lexicographic and first-fail search. Which of the two search strategeis is the best?

- Repeat the analysis on this other instance of the problem:

  $$\texttt{ten} + \texttt{ten} + \texttt{forty} = \texttt{sixty}$$

  Is the conclusion the same as in the point above?