

DM545 – Linear and integer programming

Sheet 0, Spring 2018 [pdf format]

This assignment is highly recommended for students enrolled in DM545. Students of DM559 do not need to do it.

The assignment is meant to be a review of elements of Python Programming and Linear Algebra that will be needed in Linear Programming. You can refresh or deepen your knowledge of Python with this series of tutorials:

<https://www.youtube.com/playlist?list=PL-osiE80TeTt2d9bfVyTiXJA-UTHn6WwU>

Probably, the easiest solution for you is to install Python via Anaconda. We will then need the packages `numpy` and `gurobipy`. You need to have a working installation of Python (or Anaconda) and `numpy` for Tuesday, March 20.

It is also strongly recommended that for the class on Tuesday you have read the following tutorial on `numpy` and Linear Algebra:

<http://www.imada.sdu.dk/~marco/DM559/Resources/Ipython/Tutorial.html>.

Exercise 0

Write a list of the first 100 numbers in which any number divisible by three is replaced by the word “fizz” and any divisible by five by the word “buzz”. Numbers divisible by both become “fizz buzz”.

Exercise 0 Data Types

Revise the difference between the main data types in Python: list, tuples, dictionaries and sets. Write an example for each of them in which you define and initialize a variable for each type and then print the content looping through the elements of the variable.

Exercise 1 Python: One liner quizzes

Write a one line Python code for the following tasks:

- Construct the set $S = \{x \in \mathbb{R} \mid x \geq 0 \wedge x \bmod 3 \equiv 1\}$
- Using list comprehension make a list for $\{(i, j) \mid i \in \{1, 2, 3, 4\}, j \in \{5, 7, 9\}\}$
- Calculate the inverse of a function or the index function for an invertible function (ie, bijective = injective + surjective) given in form of a dictionary.
- What is the result of the following lines?

```
map(lambda x: x%3, range(5))  
filter(lambda x: x%2==0, range(5))
```

(In Python 3.x, you have to enclose those lines in the list constructor `list()`.)

Exercise 2 Matrix Calculus in basic Python

The basic data structures in Python are lists, tuples, sets and dictionaries. Vectors and matrices can be implemented in Python as lists. How?

- Generate a couple of numerical examples for vectors and matrices. Experiment with the operators $+$ and $*$. Do they yield the same result as expected from linear algebra?
- Write a function for the sum of two vectors using list comprehension.
- Write a function for the multiplication of a vector by a scalar.
- Write a function for the sum of two matrices using list comprehension.
- Write a function for the multiplication of a matrix by a scalar.
- Write a function for the multiplication of two matrices not necessarily square. (Raise a `ValueError` exception if the size of the matrices is not compliant.)

Exercise 3 Matrix Calculus in `numpy` and `scipy`

The modules `numpy` and `scipy` make available another data structure in Python, the 'array' type. This exercise guides you to the discovery of how operators are overloaded for the 'array' type module. You can read more about `numpy` and `scipy` from the Tutorial linked above and from the section "Literature: Links" of the course web page.

Generate in Python two matrices A and B of size 3×2 and 2×4 , respectively, made of integers numbers randomly drawn from the interval $[1, \dots, 10]$. Calculate the following results, first by hand and then checking the correctness of your answer in Python:

- $A + B, A - B$
- $A \cdot B$
- A/B

[In IPython and Jupyter it is possible from command line to ask for completion via tab. This can be used to explore which functions are available for a given module. Try for example to type

```
import numpy as np
np.
```

followed by a tab. You should see a list of available functions. Among them there are two submodules that will be useful for us: `random` and `linalg`. The first implements a function to generate random numbers and matrices. The second implements functions from linear algebra. It is possible to get a manual for each function by following the function with a question mark. For example: `np.random.randint?.`]

Exercise 4

Solve by Gaussian elimination the following system of linear equations $Ax = b$ where

$$A = \begin{bmatrix} 3 & 1 & 1 & 1 \\ 2 & 4 & 1 & 1 \\ 2 & 1 & 2 & 2 \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

First carry out the calculations by hand and then try using Python.

Exercise 5

Write a one line description of the methods you know to compute the inverse of a square matrix.

Exercise 6

Calculate by hand the inverse of

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 0 & 3 \\ 0 & 0 & 2 \end{bmatrix}$$

and check your result with the function `numpy.linalg.inv`.

Exercise 7

Use Cramer's rule to express the solution of the system $Ax = \mathbf{b}$ where

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 0 & 3 \\ 0 & 0 & 2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$$

Exercise 8

A total of \$6,300 was invested in two accounts. Part was invested in obligations at 0.045 annual interest rate and part was invested in a money market fund at 0.0375 annual interest rate. If the total simple interest for one year was \$267.75, then how much was invested in each account?

Exercise 9

A construction company produces five different products: p_1, p_2, p_3, p_4, p_5 , using five resources: metal, concrete, plastic, water, electricity.

The amount of resources consumed for producing one unit of products is described by the following python dictionaries:

```
p_1={"metal":0, "concrete":1.3, "plastic":0.2, "water":.8, "electricity":.4}
p_2={"metal":0, "concrete":0, "plastic":1.5, "water":.4, "electricity":.3}
p_3={"metal":.25, "concrete":0, "plastic":0, "water":.2, "electricity":.7}
p_4={"metal":0, "concrete":0, "plastic":.3, "water":.7, "electricity":.5}
p_5={"metal":1.5, "concrete":0, "plastic":.5, "water":.4, "electricity":.8}
```

How much metal is consumed if the company decides to produce the following mix of products: 10, 9, 12, 13, 11? Express the calculation asked by the exercise as a linear algebra operation involving matrices and vectors. Then calculate the numerical answer using Python and numpy. (For example, you can transform the dictionary above into a numpy array as follows: `a=np.array(list(p_1.values()))`.)

Exercise 10

Consider again our construction company that produces five different products: p_1, p_2, p_3, p_4, p_5 , using five resources: metal, concrete, plastic, water, electricity.

The amount of resources consumed for producing one unit of products is described by the python dictionaries defined in the previous exercise.

Suppose now that you do not know how much of each product is produced but that you know how much of each material is used. Can you determine from this information the amount of each product that is produced? Will the solution be unique?

Exercise 11

Given two points in the Cartesian plane \mathbb{R}^2 , $A = (1, 2)$ and $B = (3, 4)$ write the vector parametric equation and the Cartesian equation of the line that passes through them. Express the vector equation as an affine combination of the two points.

Exercise 12

Express the segment in \mathbb{R}^2 between the points $A = (1, 2)$ and $B = (3, 4)$ as a convex combination of its extremes.

Exercise 13

Write a generic vector parametric equation and a generic Cartesian equation of a plane in \mathbb{R}^3 .

Exercise 14

Write a generic Cartesian equation of an hyperplane in \mathbb{R}^n that does not pass through the origin.

Exercise 15

Prove that the following vectors in \mathbb{R}^3 linearly independent?

- $[6, 9, 5]^T$

- $[5, 5, 7]^T$

- $[2, 0, 7]^T$