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DM559/DM545 – Linear and integer programming

Sheet 10, Spring 2018 [pdf format]

Exercise 1*

Given the Network in Figure 1, determine the max flow and indicate the min cut.

Exercise 2*

Solve the following IP problem with Gomory's fractional cutting plane algorithm, indicating the cut inequalities in the space of the original variables

$$\begin{aligned} \max \quad & x_1 + 2x_2 \\ & x_1 - 2x_2 \geq -2 \\ & x_1 + x_2 \leq 3 \\ & x_1, x_2 \geq 0 \text{ and integer} \end{aligned}$$

Exercise 3*

This exercise is taken from the exam of 2012.
The Danish Research Council has to decide which research projects to finance. The total budget for the projects is 20 million Dkk. The table below shows the evaluation from 0 (worst) to 2 (best) that the projects received by the external reviewers and the amount of money required.

	1	2	3	4	5
Evaluation score	1	1.8	1.4	0.6	1.4
Investment (in million of DKK)	6	12	10	4	8

Projects 2 and 3 have the same coordinator and the Council decided to grant only one of the two.
The Council wants to select the combination of projects that will maximize the total relevance of the projects, that is, the sum of the evaluation score while remaining within the budget.

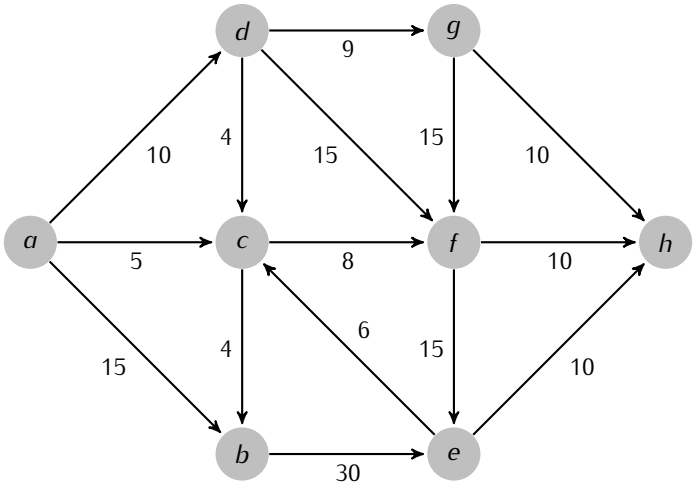

Figure 1: Find the maximum flow from a to h. Numbers on arcs are capacity values.

Formulate the problem of deciding on which project the Council has to invest as an integer linear programming problem P.

We want the IP instance solved using the branch-and-bound algorithm. What is the optimal solution x^* to the LP relaxation P'? (Hint: use Gurobi Python to find out.]

The rounding heuristic applied to the solution x^* gives a feasible solution x'. Which one? With the knowledge collected until this stage which of the three following statements is correct:

1. x' is certainly optimal
2. x' is certainly not optimal
3. x' might be optimal

(Remember to justify your answer.)

The two subproblems generated by the branch-and-bound algorithm after finding x^* correspond to choosing or not choosing a particular project. Which one?

Suppose the branch-and-bound algorithm considers first the subproblem corresponding to not choosing this project. Let's call this subproblem and its corresponding node in the search tree SP1. What is the optimal solution to its LP relaxation?

Next, the branch-and-bound algorithm considers the subproblem corresponding to choosing the project, i.e., subproblem SP2. Find the optimal solution to its LP relaxation. Which are the active nodes (i.e., open subproblems) at this point?

How does the branch and bound end?