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DM559/DM545 – Linear and integer programming

Reexam for Obligatory Assignments, Spring 2018 [pdf format]

Deadline: Sunday, August 12, 2018 at 23:59.

- This is the reexam for the obligatory assignments in DM559/DM545.
- **Students of DM559 have to do all exercises. Students of DM545 have to do only exercises 6 and 7.**
- In red the modifications after publication.
- For you answers, use the template available at this link: http://www.imada.sdu.dk/~marco/DM559/Assignments-LA/template_answers.tex
- Hand in your PDF file in BlackBoard in SDU Assignment
- The deliverable is a PDF document, as it will be at the written reexam, planned for the 17th of August. Hence, experiment and get acquainted with the tools and forms you want to use for producing this document in a similar setting as the written reexam.

  You can handwrite your answers and add them as picture in the template. Be aware that at the exam you can use digital pen or hand scanner (that is, a silent scanner) but you cannot bring handycameras. You can of course also typset your answers in LaTeX. You can use either Danish or English. Keep the newpage separation after each exercise present in the template. Write your name and CPR number where indicated in the template. In some parts, you are asked to write Python code. You have to include the code in the PDF document together with the output of the execution of the code. Use the LaTeX environment lstlisting as shown in the template for doing this. Your code must work correctly if fully copied and pasted from your report. In your answers, you have to justify the steps that you are doing. If theorems and definitions from the slides of the course or from the text book (specify which edition) are used, give reference.
- You are expected to work individually on the assignment. However, you are welcome to ask Marco for explanations that could put you on the right path for solving the tasks but it is preferable that you visit him in his office for this. He will not be avbailable for answers in the days from 9th to 12th of August.
- Failure to pass this assignment will imply the non admission to the written reexam on August 17.
- Deadline extensions will not be conceded because the written reexam is too close and cannot be postponed.

Exercise 1

From the text book [AR] (you find the references on the course web page: Do exercises:

1. 22 on page 76 (chap. 1)
2. 32 on page 77 (chap. 1)
3. 86 on page 113 (chap. 2)
4. 114 on page 165 (chap. 3)
5. 126 on page 165 (chap. 3)

Exercise 2

Consider the bases $U = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ and $V = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ for $\mathbb{R}^3$, given by the columns of the two matrices created by the following python code:

```
import numpy as np
np.random.seed(5)
U = np.random.randint(-10,10,size=(3,3))
V = np.random.randint(0,10,size=(3,3))
b = np.ones(3)
```

a. Find the transition matrix from U to V

b. Compute the coordinate vector $[\mathbf{b}]_U$, where the vector $\mathbf{b}$ with respect to the standard basis is given by the python code above.

c. Compute $[\mathbf{b}]_V$ by using $[\mathbf{b}]_V = P_{U \to V}[\mathbf{b}]_U$ and check your work by computing $[\mathbf{b}]_V$ directly from the standard basis.

Exercise 3

In each case below, find a single matrix that performs the indicated succession of operations in $\mathbb{R}^2$:

a. Expands by a factor of 3 in the x-direction, then compresses by a factor of $\frac{1}{3}$ in the y-direction.

b. Expands by a factor of 2 in the x-direction, then shears by a factor of 3 in the y-direction.

c. Rotates about $y = x$, then roates by an angle of 90° about the origin.

Consider well the order of operations and remember that matrix multiplication is not a commutative operation.

Exercise 4

Find the eigenvalues of the matrix

$$\begin{bmatrix} 5 & 0 & 1 \\ 1 & 1 & 0 \\ -7 & 1 & 0 \end{bmatrix}$$

and determine their algebraic and geometric multiplicity.

Exercise 5

Find a matrix P that diagonalizes A, and find $P^{-1}AP$.

$$A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Exercise 6 Production Planning

A firm is planning the production of 3 products A_1, A_2, A_3. In a month, the production can be active for 22 days. In the following tables are given: maximum demands (units=100 Kg), price ($/100 Kg), production costs (per 100 Kg of product), and production quotas (maximum amount of 100 Kg units of product that would be produced in a day if all production lines were dedicated to the product).

Product	A_1	A_2	A_3
Maximum demand	5300	4500	5400
Selling price	$124	$109	$115
Production cost	$73.30	$52.90	$65.40
Production quota	500	450	550

a. Formulate a mathematical programming model to determine the production plan to maximize the total income.

b. Solve the model in Gurobi and Python on the given data.

c. Change the mathematical program to account for a fixed activation cost on the production line, as follows:

Product	A_1	A_2	A_3
Activation cost	$170000	$150000	$100000

d. Solve the new model in Gurobi and Python on the given data.

e. Change the mathematical program and the AMPL model to cater for both the fixed activation cost and for a minimum production batch:

Product	A_1	A_2	A_3
Minimum batch	20	20	16

f. Solve also this last model in Gurobi and Python on the given data.

Exercise 7 Parking

On Dantzig Street cars can be parked on both sides of the street. Mr. Gomory, who lives at number 1, is organizing a party for around 30 people, who will arrive in 15 cars. The length of the ith car, $i = 1, \dots, 15$, is λ_i expressed in meters as follows:

```
lengths = [4, 4.5, 5, 4.1, 2.4, 5.2, 3.7, 3.5, 3.2, 4.5, 2.3, 3.3, 3.8,
  4.6, 3]
```

In order to avoid bothering the neighbors, Mr. Gomory would like to arrange the parking on both sides of the street so that the length of the street occiped by his friends' cars should be minimum.

a. Give a mathematical programming formulation for a general instance of the problem.

b. Solve the problem in Gurobi using Python. Write the code in implicit form, that is, separating the model from the data.

c. Report the results.

d. How does the model change if on exactly one of the street sides the cars should not occupy more than 15 meters?