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DM545/DM871 – Linear and integer programming

Sheet 2, Spring 2020 [pdf format]

Solution:

Included.

Exercise 1

Solve the following LP problem

$$\begin{array}{ll} \text{maximize} & 10x_1 - 57x_2 - 9x_3 - 24x_4 \\ \text{subject to} & x_1 \leq 1 \\ & 1/2x_1 - 11/2x_2 - 5/2x_3 + 9x_4 \leq 0 \\ & +1/2x_1 - 3/2x_2 - 1/2x_3 + x_4 \leq 0 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{array}$$

using the following pivot rule:

i. the entering variable will always be the nonbasic variable that has the largest coefficient in the z-row of the dictionary.

ii. if two or more basic variables compete for leaving the basis, then the candidate with the smallest subscript will be made to leave.

There are quite a few calculations to carry out, hence you are reccomended to use Python. See the guidelines at: http://www.imada.sdu.dk/~marco/DM545/Resources/Ipython/Tutorial4Exam.html

Solution:

```
|--------+--------+--------+--------+--------+--------+--------+--------+--------+
|     x1 |     x2 |     x3 |     x4 |     x5 |     x6 |     x7 |     -z |      b |
|--------+--------+--------+--------+--------+--------+--------+--------+--------+
|    1/2 |  -11/2 |   -5/2 |      9 |      1 |      0 |      0 |      0 |      0 |
|    1/2 |   -3/2 |   -1/2 |      1 |      0 |      1 |      0 |      0 |      0 |
|      1 |      0 |      0 |      0 |      0 |      0 |      1 |      0 |      1 |
|--------+--------+--------+--------+--------+--------+--------+--------+--------+
|     10 |    -57 |     -9 |    -24 |      0 |      0 |      0 |      1 |      0 |
|--------+--------+--------+--------+--------+--------+--------+--------+--------+
PRIMAL SIMPLEX

pivot column: 1
pivot row: 1
 pivot: 1/2


|--------+--------+--------+--------+--------+--------+--------+--------+--------+
|     x1 |     x2 |     x3 |     x4 |     x5 |     x6 |     x7 |     -z |      b |
|--------+--------+--------+--------+--------+--------+--------+--------+--------+
|      1 |    -11 |     -5 |     18 |      2 |      0 |      0 |      0 |      0 |
|      0 |      4 |      2 |     -8 |     -1 |      1 |      0 |      0 |      0 |
|      0 |     11 |      5 |    -18 |     -2 |      0 |      1 |      0 |      1 |
|--------+--------+--------+--------+--------+--------+--------+--------+--------+
|      0 |     53 |     41 |   -204 |    -20 |      0 |      0 |      1 |      0 |
```

```
|--------+--------+--------+--------+--------+--------+--------+--------+--------+

pivot column: 2
pivot row: 2
 pivot: 4

|--------+--------+--------+--------+--------+--------+--------+--------+--------+
|     x1 |     x2 |     x3 |     x4 |     x5 |     x6 |     x7 |     -z |      b |
|--------+--------+--------+--------+--------+--------+--------+--------+--------+
|      1 |      0 |    1/2 |     -4 |   -3/4 |   11/4 |      0 |      0 |      0 |
|      0 |      1 |    1/2 |     -2 |   -1/4 |    1/4 |      0 |      0 |      0 |
|      0 |      0 |   -1/2 |      4 |    3/4 |  -11/4 |      1 |      0 |      1 |
|--------+--------+--------+--------+--------+--------+--------+--------+--------+
|      0 |      0 |   29/2 |    -98 |  -27/4 |  -53/4 |      0 |      1 |      0 |
|--------+--------+--------+--------+--------+--------+--------+--------+--------+

pivot column: 3
pivot row: 1
 pivot: 1/2

|--------+--------+--------+--------+--------+--------+--------+--------+--------+
|     x1 |     x2 |     x3 |     x4 |     x5 |     x6 |     x7 |     -z |      b |
|--------+--------+--------+--------+--------+--------+--------+--------+--------+
|      2 |      0 |      1 |     -8 |   -3/2 |   11/2 |      0 |      0 |      0 |
|     -1 |      1 |      0 |      2 |    1/2 |   -5/2 |      0 |      0 |      0 |
|      1 |      0 |      0 |      0 |      0 |      0 |      1 |      0 |      1 |
|--------+--------+--------+--------+--------+--------+--------+--------+--------+
|    -29 |      0 |      0 |     18 |     15 |    -93 |      0 |      1 |      0 |
|--------+--------+--------+--------+--------+--------+--------+--------+--------+

pivot column: 4
pivot row: 2
 pivot: 2

|--------+--------+--------+--------+--------+--------+--------+--------+--------+
|     x1 |     x2 |     x3 |     x4 |     x5 |     x6 |     x7 |     -z |      b |
|--------+--------+--------+--------+--------+--------+--------+--------+--------+
|     -2 |      4 |      1 |      0 |    1/2 |   -9/2 |      0 |      0 |      0 |
|   -1/2 |    1/2 |      0 |      1 |    1/4 |   -5/4 |      0 |      0 |      0 |
|      1 |      0 |      0 |      0 |      0 |      0 |      1 |      0 |      1 |
|--------+--------+--------+--------+--------+--------+--------+--------+--------+
|    -20 |     -9 |      0 |      0 |   21/2 | -141/2 |      0 |      1 |      0 |
|--------+--------+--------+--------+--------+--------+--------+--------+--------+

pivot column: 5
pivot row: 1
 pivot: 1/2

|--------+--------+--------+--------+--------+--------+--------+--------+--------+
|     x1 |     x2 |     x3 |     x4 |     x5 |     x6 |     x7 |     -z |      b |
|--------+--------+--------+--------+--------+--------+--------+--------+--------+
|     -4 |      8 |      2 |      0 |      1 |     -9 |      0 |      0 |      0 |
|    1/2 |   -3/2 |   -1/2 |      1 |      0 |      1 |      0 |      0 |      0 |
|      1 |      0 |      0 |      0 |      0 |      0 |      1 |      0 |      1 |
|--------+--------+--------+--------+--------+--------+--------+--------+--------+
|     22 |    -93 |    -21 |      0 |      0 |     24 |      0 |      1 |      0 |
|--------+--------+--------+--------+--------+--------+--------+--------+--------+
```

```
pivot column: 6
pivot row: 2
 pivot: 1

|--------+--------+--------+--------+--------+--------+--------+--------+--------+
|     x1 |     x2 |     x3 |     x4 |     x5 |     x6 |     x7 |     -z |      b |
|--------+--------+--------+--------+--------+--------+--------+--------+--------+
|    1/2 |  -11/2 |   -5/2 |      9 |      1 |      0 |      0 |      0 |      0 |
|    1/2 |   -3/2 |   -1/2 |      1 |      0 |      1 |      0 |      0 |      0 |
|      1 |      0 |      0 |      0 |      0 |      0 |      1 |      0 |      1 |
|--------+--------+--------+--------+--------+--------+--------+--------+--------+
|     10 |    -57 |     -9 |    -24 |      0 |      0 |      0 |      1 |      0 |
|--------+--------+--------+--------+--------+--------+--------+--------+--------+

pivot column: 1
pivot row: 1
 pivot: 1/2
```

Thus we discover that we return to the first tableau and that therefore we are cycling. We are in a malignous degenerancy. In order to make it benignous, that is, in order to avoid cycling a different pivoting rule must be used. The sign that we are in a degenerate case that might turn out malignous is the fact that one of the b_i terms is zero. This implies that there is a basic variable that gets value zero.

Exercise 2

Solve the following problem, known as the Klee-Minty problem, using the largest coefficient pivoting rule.

$$\begin{array}{ll} \text{maximize} & 100x_1 + 10x_2 + x_3 \\ \text{subject to} & x_1 \leq 1 \\ & 20x_1 + x_2 \leq 100 \\ & 200x_1 + 20x_2 + x_3 \leq 10000 \\ & x_1, x_2 \geq 0 \end{array}$$

Can you generalize the example to n variables and guess what will be the number of iterations the simplex will do?

Solution:

```
|--------+--------+--------+--------+--------+--------+--------+--------+
|     x1 |     x2 |     x3 |     x4 |     x5 |     x6 |     -z |      b |
|--------+--------+--------+--------+--------+--------+--------+--------+
|      1 |      0 |      0 |      1 |      0 |      0 |      0 |      1 |
|     20 |      1 |      0 |      0 |      1 |      0 |      0 |    100 |
|    200 |     20 |      1 |      0 |      0 |      1 |      0 |  10000 |
|--------+--------+--------+--------+--------+--------+--------+--------+
|    100 |     10 |      1 |      0 |      0 |      0 |      1 |      0 |
|--------+--------+--------+--------+--------+--------+--------+--------+
PRIMAL SIMPLEX

pivot column: 1
pivot row: 1
 pivot: 1
1

|--------+--------+--------+--------+--------+--------+--------+--------+
|     x1 |     x2 |     x3 |     x4 |     x5 |     x6 |     -z |      b |
|--------+--------+--------+--------+--------+--------+--------+--------+
|      1 |      0 |      0 |      1 |      0 |      0 |      0 |      1 |
```

```
|      0 |      1 |      0 |    -20 |      1 |      0 |      0 |     80 |
|      0 |     20 |      1 |   -200 |      0 |      1 |      0 |   9800 |
|--------+--------+--------+--------+--------+--------+--------+--------+
|      0 |     10 |      1 |   -100 |      0 |      0 |      1 |   -100 |
|--------+--------+--------+--------+--------+--------+--------+--------+

pivot column: 2
pivot row: 2
 pivot: 1
1

|--------+--------+--------+--------+--------+--------+--------+--------+
|     x1 |     x2 |     x3 |     x4 |     x5 |     x6 |     -z |      b |
|--------+--------+--------+--------+--------+--------+--------+--------+
|      1 |      0 |      0 |      1 |      0 |      0 |      0 |      1 |
|      0 |      1 |      0 |    -20 |      1 |      0 |      0 |     80 |
|      0 |      0 |      1 |    200 |    -20 |      1 |      0 |   8200 |
|--------+--------+--------+--------+--------+--------+--------+--------+
|      0 |      0 |      1 |    100 |    -10 |      0 |      1 |   -900 |
|--------+--------+--------+--------+--------+--------+--------+--------+

pivot column: 4
pivot row: 1
 pivot: 1
1

|--------+--------+--------+--------+--------+--------+--------+--------+
|     x1 |     x2 |     x3 |     x4 |     x5 |     x6 |     -z |      b |
|--------+--------+--------+--------+--------+--------+--------+--------+
|      1 |      0 |      0 |      1 |      0 |      0 |      0 |      1 |
|     20 |      1 |      0 |      0 |      1 |      0 |      0 |    100 |
|   -200 |      0 |      1 |      0 |    -20 |      1 |      0 |   8000 |
|--------+--------+--------+--------+--------+--------+--------+--------+
|   -100 |      0 |      1 |      0 |    -10 |      0 |      1 |  -1000 |
|--------+--------+--------+--------+--------+--------+--------+--------+

pivot column: 3
pivot row: 3
 pivot: 1
1

|--------+--------+--------+--------+--------+--------+--------+--------+
|     x1 |     x2 |     x3 |     x4 |     x5 |     x6 |     -z |      b |
|--------+--------+--------+--------+--------+--------+--------+--------+
|      1 |      0 |      0 |      1 |      0 |      0 |      0 |      1 |
|     20 |      1 |      0 |      0 |      1 |      0 |      0 |    100 |
|   -200 |      0 |      1 |      0 |    -20 |      1 |      0 |   8000 |
|--------+--------+--------+--------+--------+--------+--------+--------+
|    100 |      0 |      0 |      0 |     10 |     -1 |      1 |  -9000 |
|--------+--------+--------+--------+--------+--------+--------+--------+

pivot column: 1
pivot row: 1
 pivot: 1
1

|--------+--------+--------+--------+--------+--------+--------+--------+
|     x1 |     x2 |     x3 |     x4 |     x5 |     x6 |     -z |      b |
```

```
|--------+--------+--------+--------+--------+--------+--------+--------+
|      1 |      0 |      0 |      1 |      0 |      0 |      0 |      1 |
|      0 |      1 |      0 |    -20 |      1 |      0 |      0 |     80 |
|      0 |      0 |      1 |    200 |    -20 |      1 |      0 |   8200 |
|--------+--------+--------+--------+--------+--------+--------+--------+
|      0 |      0 |      0 |   -100 |     10 |     -1 |      1 |  -9100 |
|--------+--------+--------+--------+--------+--------+--------+--------+

pivot column: 5
pivot row: 2
 pivot: 1
1

|--------+--------+--------+--------+--------+--------+--------+--------+
|     x1 |     x2 |     x3 |     x4 |     x5 |     x6 |     -z |      b |
|--------+--------+--------+--------+--------+--------+--------+--------+
|      1 |      0 |      0 |      1 |      0 |      0 |      0 |      1 |
|      0 |      1 |      0 |    -20 |      1 |      0 |      0 |     80 |
|      0 |     20 |      1 |   -200 |      0 |      1 |      0 |   9800 |
|--------+--------+--------+--------+--------+--------+--------+--------+
|      0 |    -10 |      0 |    100 |      0 |     -1 |      1 |  -9900 |
|--------+--------+--------+--------+--------+--------+--------+--------+

pivot column: 4
pivot row: 1
 pivot: 1
1

|--------+--------+--------+--------+--------+--------+--------+--------+
|     x1 |     x2 |     x3 |     x4 |     x5 |     x6 |     -z |      b |
|--------+--------+--------+--------+--------+--------+--------+--------+
|      1 |      0 |      0 |      1 |      0 |      0 |      0 |      1 |
|     20 |      1 |      0 |      0 |      1 |      0 |      0 |    100 |
|    200 |     20 |      1 |      0 |      0 |      1 |      0 |  10000 |
|--------+--------+--------+--------+--------+--------+--------+--------+
|   -100 |    -10 |      0 |      0 |      0 |     -1 |      1 | -10000 |
|--------+--------+--------+--------+--------+--------+--------+--------+
```