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AI505 – Optimization

Sheet 06, Spring 2025

Exercises with the symbol + are to be done at home before the class. Exercises with the symbol * will be tackled in class. The remaining exercises are left for self training after the exercise class. Some exercises are from the text book and the number is reported. They have the solution at the end of the book.

Exercise 1+ (11.1)

Suppose you do not know any optimization algorithm for solving a linear program. You decide to evaluate all the vertices and determine, by inspection, which one minimizes the objective function. Give a loose upper bound on the number of possible minimizers you will examine. Furthermore, does this method properly handle all linear constrained optimization problems?

Exercise 2+ (11.2)

If the program in example 11.1 is bounded below, argue that the simplex method must converge.

Exercise 3+ (11.3)

Suppose we want to solve:

$$\begin{aligned} \text{minimize} \;\; & 6x_1 + 5x_2 \\ \text{subject to} \;\; & 3x_1 - 2x_2 \geq 5. \end{aligned}$$

How would we translate this problem into a linear program in equality form with the same minimizer?

Exercise 4

Consider the following problem:

$$\begin{aligned} \text{minimize} \; & 5x_1 + 4x_2 \\ \text{s.t.} \; & 2x_1 + 3x_2 \leq 5 \\ & 4x_1 + x_2 \leq 11 \end{aligned}$$

Solve the problem numerically implementing the simplex algorithm.

Exercise 5+ (11.4)

Suppose your optimization algorithm has found a search direction $\boldsymbol{d}$ and you want to conduct a line search. However, you know that there is a linear constraint $\boldsymbol{w}^T\boldsymbol{x} \geq \mathbf{0}$. How would you modify the line search to take this constraint into account? You can assume that your current design point is feasible.

Exercise 6+ (11.5)

Reformulate the linear program

$$\begin{aligned} \text{minimize} \; & \boldsymbol{c}^T\boldsymbol{x} \\ \text{s.t.} \; & A\boldsymbol{x} \geq 0 \end{aligned}$$

into an unconstrained optimization problem with a log barrier penalty.

Exercise 7*

Drug	A	B	C		
raw material (Kg)	5	8	6		600
processing time (hours)	3	4	5		400
packaging units	2	3	1		200
profit	60	100	80	-5	

Table 1: Data from the pharmaceutical company

A pharmaceutical company produces three types of drugs: A, B, and C. These drugs require raw materials, chemical processing time, and packaging units. The goal is to maximize profit, taking into account restrictions due to resource availability and production balance.
Drug A contributes 60 DKK per unit to the profit, drug B contributes 100 DKK per unit, drug C contributes 80 DKK per unit. The consumptions per units of drug of raw materials, chemical processing time, and packaging units are given in Table **??** together with the quantities available. While consumptions of raw material and processing time cannot exceed the amount available, excess of packaging units is allowed. Extra packaging units can be bought but each extra unit reduces profit at 5 DKK per unit while packaging units left can be reused in the next production period and hence contribute positively to the profit with the same amount of DKK per unit. Due to contractual obligations, Drug B must be produced in exactly twice the amount of Drug A.
Formulate the problem in linear programming terms. Write first the *instantiated* model and then the abstract, *general* model separating model from data.

Exercise 8*

Consider the following linear programming problem:

$$
\begin{aligned}
\text{maximize } Z = 60x_1 + 100x_2 + 80x_3 - 5x_4 \\
\text{s.t. } 5x_1 + 8x_2 + 6x_3 \leq 600 \\
3x_1 + 4x_2 + 5x_3 \leq 400 \\
2x_1 + 3x_2 + x_3 = 200 + x_4 \\
x_2 = 2x_1 \\
x_1, x_2, x_3 \geq 0 \\
x_4 \in \mathbb{R}
\end{aligned}
$$

Your tasks:

- Transform the problem in standard form
- Transform the problem in equality form
- Solve the problem numerically with `scipy.optimize.linprog`. Read this tutorial.