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DM811 - Heuristics for Combinatorial Optimization

Laboratory Assignment 3, Fall 2008

Prepare at least two of the exercises below to discuss in class. You may work in group, if you prefer.

Exercise 1

Definition. Bin Packing Problem
Input: *A finite set U of items, a size $s(u) \in Z^+$ for each $u \in U$, and a positive integer bin capacity B.*
Task: *Find the minimal number of bins K for which there exits a partition of U into disjoint sets $U_1, U_2, \ldots, U_k$ and the sum of the sizes of the items in each U_i is B or less.*

Definition. Two-dimensional bin packing
Input: *A finite set U of rectangular items, each with a width $w_u \in Z^+$ and a height $h_u \in Z^+$, $u \in U$, and an unlimited number of identical rectangular bins of width $W \in Z^+$ and height $H \in Z^+$.*
Task: *Allocate all the items into a minimum number of bins, such that the bin widths and heights are not exceeded and the original orientation is respected (no rotation of the items is allowed).*

Design a simple construction heuristic and a simple local search algorithm for the two problems.

Exercise 2

We recall the definition of the SMTWTP given in lecture

Definition. Single Machine Total Weighted Tardiness Problem
Input: *A set J of jobs $\{1, \ldots, n\}$ to be processed on a single machine and for each job $j \in J$ a processing time p_j, a weight w_j and a due date d_j.*

Task: *Find a schedule that minimizes the total weighted tardiness $\sum_{j=1}^{n} w_j \cdot T_j$, where $T_j = \{C_j - d_j, 0\}$ (C_j completion time of job j).*

Give a computational analysis for a local search with the following three neighborhoods for local search: interchange, insertion, swap. Discuss possible neighborhood pruning and show that the insert neighborhood can be evaluated in $O(n^2)$.

Exercise 3

Definition. p-Median Problem
Input: *A set U of locations for n* users, *a set F of locations for m* facilities *and a distance matrix $D = [d_{ij}] \in \mathbf{R}^{n \times m}$.*

Task: *Select a set $J \subseteq F$ of p locations where to install facilities such that the sum of the distances of each user to its closest installed facility is minimized, i.e.,*

$$\min \Big\{ \sum_{i \in U} \min_{j \in J} d_{ij} \mid J \subseteq F \text{ and } |J| = p \Big\}$$

Design a simple construction heuristic and a simple local search algorithm.

Exercise 4

Definition. Quadratic Assignment Problem
Input: *A set of n* locations *with a matrix $D = [d_{ij}] \in \mathbf{R}^{n \times n}$ of distances and a set of n* units *with a matrix $F = [f_{kl}] \in \mathbf{R}^{n \times n}$ of flows between them*

Task: *Find the assignment σ of units to locations that minimizes the sum of products between flows and distances, i.e.,*

$$\min_{\sigma \in \Sigma} \sum_{i,j} f_{ij} d_{\sigma(i)\sigma(j)}$$

Define solution representation, evaluation function and neighborhood for a local search algorithm. Make a computational analysis and show that a single neighbor can be evaluated in $O(n)$.

Exercise 5

Definition. Set Problems

Set Covering

$$\begin{aligned} \min \quad & \sum_{j=1}^{n} c_j x_j \\ & \sum_{j=1}^{n} a_{ij} x_j \geq 1 \quad \forall i \\ & x_j \in \{0,1\} \end{aligned}$$

Set Partitioning

$$\begin{aligned} \min \quad & \sum_{j=1}^{n} c_j x_j \\ & \sum_{j=1}^{n} a_{ij} x_j = 1 \quad \forall i \\ & x_j \in \{0,1\} \end{aligned}$$

Set Packing

$$\begin{aligned} \max \quad & \sum_{j=1}^{n} c_j x_j \\ & \sum_{j=1}^{n} a_{ij} x_j \leq 1 \quad \forall i \\ & x_j \in \{0,1\} \end{aligned}$$

Design a simple construction heuristic and a simple local search algorithm for these problems.

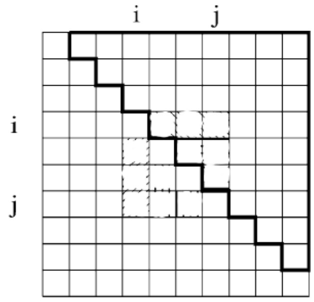
Exercise 6

Definition. Linear Ordering Problem
The two following problems are equivalent.
Input: *an $n \times n$ matrix C*

Task: *Find a permutation π of the column and row indices $\{1, \ldots, n\}$ such that the value*

$$f(\pi) = \sum_{i=1}^{n} \sum_{j=i+1}^{n} c_{\pi_i \pi_j}$$

is maximized. In other terms, find a permutation of the columns and rows of C such that the elements in the upper triangle is maximized.



Definition. Feedback arc set problem (FASP)
Input: *A directed graph $D = (V, A)$, where $V = \{1, 2, \ldots, n\}$, and arc weights c_{ij} for all $[ij] \in A$*

Task: *Find a permutation $\pi_1, \pi_2, \ldots \pi_n$ of V (that is, a linear ordering of V) such that the total costs of those arcs $[\pi_j \pi_i]$ where $j > i$ (that is, the arcs that point backwards in the ordering)*

$$f(\pi) = \sum_{i=1}^{n} \sum_{j=i+1}^{n} c_{\pi_j \pi_i}$$

is minimized.

Design a simple construction heuristic and a simple local search algorithm.

Exercise 7

At lecture we discussed the Vertex-Graph Coloring Problem in its optimization version and mentioned different solution approaches for local search. Complete the following table adding a "+" or "-" in the fourth column indicating the feasibility or infeasibility of designing a local search algorithm under the corresponding approach.

For each of the rows that you marked as possible define *candidate solutions*, *neighborhood relation* and *evaluation function*.

k	assignment	coloring	feasibility
k-fixed	complete	proper	
k-fixed	partial	proper	
k-fixed	complete	unproper	
k-fixed	partial	unproper	
k-variable	complete	proper	
k-variable	partial	proper	
k-variable	complete	unproper	
k-variable	partial	unproper	

Exercise 8

Recall the definition of the SAT problem. Indicate which auxiliary data structures can be implemented to speed up the examination of the one-flip neighborhood in a local search algorithm. Provide a computational analysis.