DM87
SCHEDULING,
TIMETABLING AND ROUTING

Lecture 12

Job Shop and Resource Constrained Project Scheduling

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Outline

Resume

Flow Shop

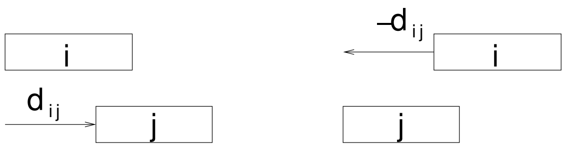
- Iterated Greedy
- Tabu Search (block representation and neighborhood pruning)

Job Shop:

- Definition
- Starting times and m-tuple permutation representation
- Disjunctive graph representation [Roy and Sussman, 1964]
- Shifting Bottleneck Heuristic [Adams, Balas and Zawack, 1988]

Outline

Generalizations: Time Lags



Generalized time constraints

They can be used to model:

- Release time:

$$S_0 + r_i \leq S_i \quad \Longleftrightarrow \quad d_{0i} = r_i$$

- Deadlines:

$$S_i + p_i - d_i \leq S_0 \quad \Longleftrightarrow \quad d_{i0} = p_i - d_i$$

- Modelling

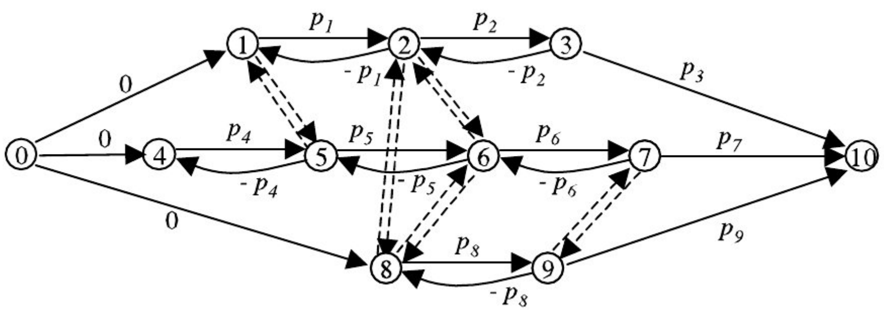
$$\begin{array}{lll} \min & C_{max} & \\ \text{s.t.} & x_{ij} + d_{ij} \leq C_{max} & \forall\, O_{ij} \in N \\ & x_{ij} + d_{ij} \leq x_{lj} & \forall\, (O_{ij}, O_{lj}) \in A \\ & x_{ij} + d_{ij} \leq x_{ik} \vee x_{ij} + d_{ij} \leq x_{ik} & \forall\, (O_{ij}, O_{ik}) \in E \\ & x_{ij} \geq 0 & \forall\, i = 1, \ldots, m \;\; j = 1, \ldots, N \end{array}$$

- In the disjunctive graph, d_{ij} become the lengths of arcs

- Exact relative timing (perishability constraints):
  if operation j must start l_{ij} after operation i:

$$S_i + p_i + l_{ij} \leq S_j \quad \text{and} \quad S_j - (p_i + l_{ij}) \leq S_i$$

($l_{ij} = 0$ if no-wait constraint)



- Set up times:

$$S_i + p_i + s_{ij} \leq S_j \quad \text{or} \quad S_j + p_j + s_{ji} \leq S_i$$

- Machine unavailabilities:
  - Machine M_k unavailable in $[a_1, b_1]$, $[a_2, b_2]$, ... , $[a_v, b_v]$
  - Introduce v artificial operations with $\lambda = 1, \ldots, v$ with $\mu_\lambda = M_k$ and:
    $p_\lambda = b_\lambda - a_\lambda$
    $r_\lambda = a_\lambda$
    $d_\lambda = b_\lambda$

- Minimum lateness objectives:

$$L_{max} = \max_{j=1}^{N}\{C_j - d_j\} \quad \Longleftrightarrow \quad d_{n_j, n+1} = p_{n_j} - d_j$$

Blocking

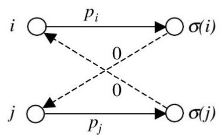
Arises with limited buffers:
after processing, a job remains on the machine until the next machine is freed

- Needed a generalization of the disjunctive graph model
  $\Longrightarrow$ Alternative graph model $G = (N, E, A)$ [Mascis, Pacciarelli, 2002]

1. two non-blocking operations to be processed on the same machine

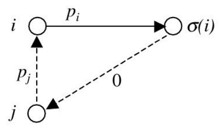
$$S_i + p_i \leq S_j \quad \text{or} \quad S_j + p_j \leq S_i$$

2. Two blocking operations i, j to be processed on the same machine $\mu(i) = \mu(j)$



$$S_{MS(j)} \leq S_i \quad \text{or} \quad S_{MS(i)} \leq S_j$$

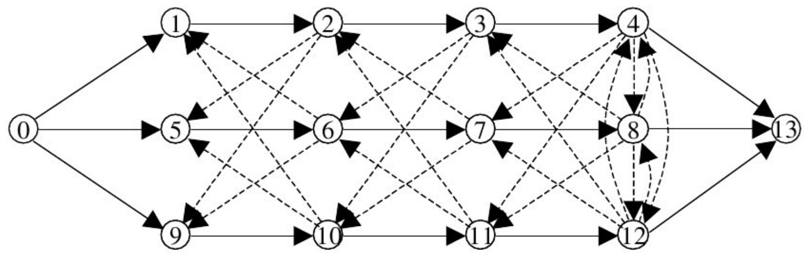
3. i is blocking, j is non-blocking (ideal) and i, j to be processed on the same machine $\mu(i) = \mu(j)$.



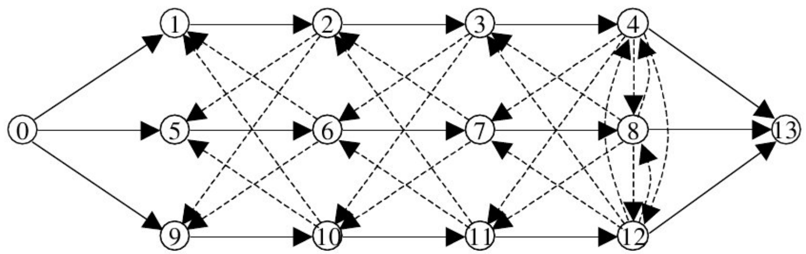
$$S_i + p_i \leq S_j \quad \text{or} \quad S_{MS(j)} \leq S_i$$

Example

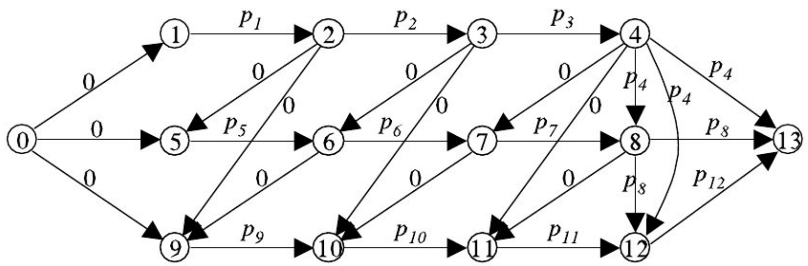
- $O_0, O_1, \ldots, O_{13}$
- $M(O_1) = M(O_5) = M(O_9)$
  $M(O_2) = M(O_6) = M(O_{10})$
  $M(O_3) = M(O_7) = M(O_{11})$



- Length of arcs can be negative
- Multiple occurrences possible: $((i,j),(u,v)) \in A$ and $((i,j),(h,k)) \in A$
- The last operation of a job j is always non-blocking.



- A complete selection S is consistent if it chooses alternatives from each pair such that the resulting graph does not contain positive cycles.



Example:

- $p_a = 4$
- $p_b = 2$
- $p_c = 1$

- b must start at least 9 days after a has started
- c must start at least 8 days after b is finished
- c must finish within 16 days after a has started

$$\begin{aligned} S_a + 9 &\leq S_b \\ S_b + 10 &\leq S_c \\ S_c - 15 &\leq S_a \end{aligned}$$

This leads to an absurd.
In the alternative graph the cycle is positive.

- The Makespan still corresponds to the longest path in the graph with the arc selection $G(S)$.
- If there are no cycles of length strictly positive it can still be computed efficiently in $O(|N||E \cup A|)$ by Bellman-Ford (1958) algorithm.
- The algorithm iteratively considers all edges in a certain order and updates an array of longest path lengths for each vertex. It stops if a loop over all edges does not yield any update or after $|N|$ iterations over all edges (in which case we know there is a positive cycle).
- Possible to maintain incremental updates when changing the selection [Demetrescu Frangioni, Marchetti-Spaccamela, Nanni, 2000].

Heuristics for the Alternative Graph Model

- The search space is highly constrained + detecting positive cycles is costly
- Hence local search methods not very successful
- Rely on the construction paradigm
- Rollout algorithm [Meloni, Pacciarelli, Pranzo, 2004]

Rollout

- Master process: grows a partial selection S^k: decides the next element to fix based on an heuristic function (selects the one with minimal value)
- Slave process: evaluates heuristically the alternative choices. Completes the selection by keeping fixed what passed by the master process and fixing one alternative at a time.

- Slave heuristics
  - *Avoid Maximum Current Completion time*
    find an arc (h, k) that if selected would increase most the length of the longest path in $G(S^k)$ and select its alternative

$$\max_{(uv)\in A} \{l(0,u) + a_{uv} + l(u,n)\}$$

  - *Select Most Critical Pair*
    find the pair that, in the worst case, would increase least the length of the longest path in $G(S^k)$ and select the best alternative

$$\max_{((ij),(hk))\in A} \min\{l(0,u) + a_{hk} + l(k,n), l(0,i) + a_{ij} + l(j,n)\}$$

  - *Select Max Sum Pair*
    finds the pair with greatest potential effect on the length of the longest path in $G(S^k)$ and select the best alternative

$$\max_{((ij),(hk))\in A} |l(0,u) + a_{hk} + l(k,n) + l(0,i) + a_{ij} + l(j,n)|$$

Trade off quality *vs* keeping feasibility
Results depend on the characteristics of the instance.

Outline

1. Job Shop Generalizations

2. Resource Constrained Project Scheduling Model

Resource Constrained Project Scheduling Model

Given:

- activities (jobs) $j = 1, \ldots, n$
- renewable resources $i = 1, \ldots, m$
- amount of resources available R_i
- processing times p_j
- amount of resource used r_{ij}
- precedence constraints $j \rightarrow k$

Further generalizations

- Time dependent resource profile $R_i(t)$ given by (t_i^μ, R_i^μ) where $0 = t_i^1 < t_i^2 < \ldots < t_i^{m_i} = T$ Disjunctive resource, if $R_k(t) = \{0, 1\}$; cumulative resource, otherwise
- Multiple modes for an activity j processing time and use of resource depends on its mode m: p_{jm}, r_{jkm}.

Solutions

Task: Find a schedule indicating the starting time of each activity

- All solution methods restrict the search to feasible schedules, S, S'
- Types of schedules
  - Local left shift (LLS): $S \rightarrow S'$ with $S'_j < S_j$ and $S'_l = S_l$ for all $l \neq j$.
  - Global left shift (GLS): LLS passing through infeasible schedule
  - Semi active schedule: no LLS possible
  - Active schedule: no GLS possible
  - Non-delay schedule: no GLS and LLS possible even with preemption
- If regular objectives $\Longrightarrow$ exists an optimum which is active

Hence:

- Schedule not given by start times S_i
  - space too large $O(T^n)$
  - difficult to check feasibility
- Sequence (list, permutation) of activities $\pi = (j_1, \ldots, j_n)$
- π determines the order of activities to be passed to a schedule generation scheme

Modeling

Assignment 1

- A contractor has to complete n activities.
- The duration of activity j is p_j
- each activity requires a crew of size W_j.
- The activities are not subject to precedence constraints.
- The contractor has W workers at his disposal
- his objective is to complete all n activities in minimum time.

Assignment 2

- Exams in a college may have different duration.
- The exams have to be held in a gym with W seats.
- The enrollment in course j is W_j and
- all W_j students have to take the exam at the same time.
- The goal is to develop a timetable that schedules all n exams in minimum time.
- Consider both the cases in which each student has to attend a single exam as well as the situation in which a student can attend more than one exam.

Assignment 3

- In a basic high-school timetabling problem we are given m classes $c_1, \ldots, c_m$,
- h teachers $a_1, \ldots, a_h$ and
- T teaching periods $t_1, \ldots, t_T$.
- Furthermore, we have lectures $i = l_1, \ldots, l_n$.
- Associated with each lecture is a unique teacher and a unique class.
- A teacher a_j may be available only in certain teaching periods.
- The corresponding timetabling problem is to assign the lectures to the teaching periods such that
  - each class has at most one lecture in any time period
  - each teacher has at most one lecture in any time period,
  - each teacher has only to teach in time periods where he is available.

Assignment 4

- A set of jobs $J_1, \ldots, J_g$ are to be processed by auditors $A_1, \ldots, A_m$.
- Job J_l consists of n_l tasks ($l = 1, \ldots, g$).
- There are precedence constraints $i_1 \to i_2$ between tasks i_1, i_2 of the same job.
- Each job J_l has a release time r_l, a due date d_l and a weight w_l.
- Each task must be processed by exactly one auditor. If task i is processed by auditor A_k, then its processing time is p_{ik}.
- Auditor A_k is available during disjoint time intervals $[s_k^\nu, l_k^\nu]$ ($\nu = 1, \ldots, m$) with $l_k^\nu < s_k^\nu$ for $\nu = 1, \ldots, m_k - 1$.
- Furthermore, the total working time of A_k is bounded from below by H_k^- and from above by H_k^+ with $H_k^- \leq H_k^+$ $(k = 1, \ldots, m)$.
- We have to find an assignment $\alpha(i)$ for each task $i = 1, \ldots, n := \sum_{l=1}^{g} n_l$ to an auditor $A_{\alpha(i)}$ such that
  - each task is processed without preemption in a time window of the assigned auditor
  - the total workload of A_k is bounded by H_k^- and H_k^k for $k = 1, \ldots, m$.
  - the precedence constraints are satisfied,
  - all tasks of J_l do not start before time r_l, and
  - the total weighted tardiness $\sum_{l=1}^{g} w_l T_l$ is minimized.